Girls' Angle Bulletin

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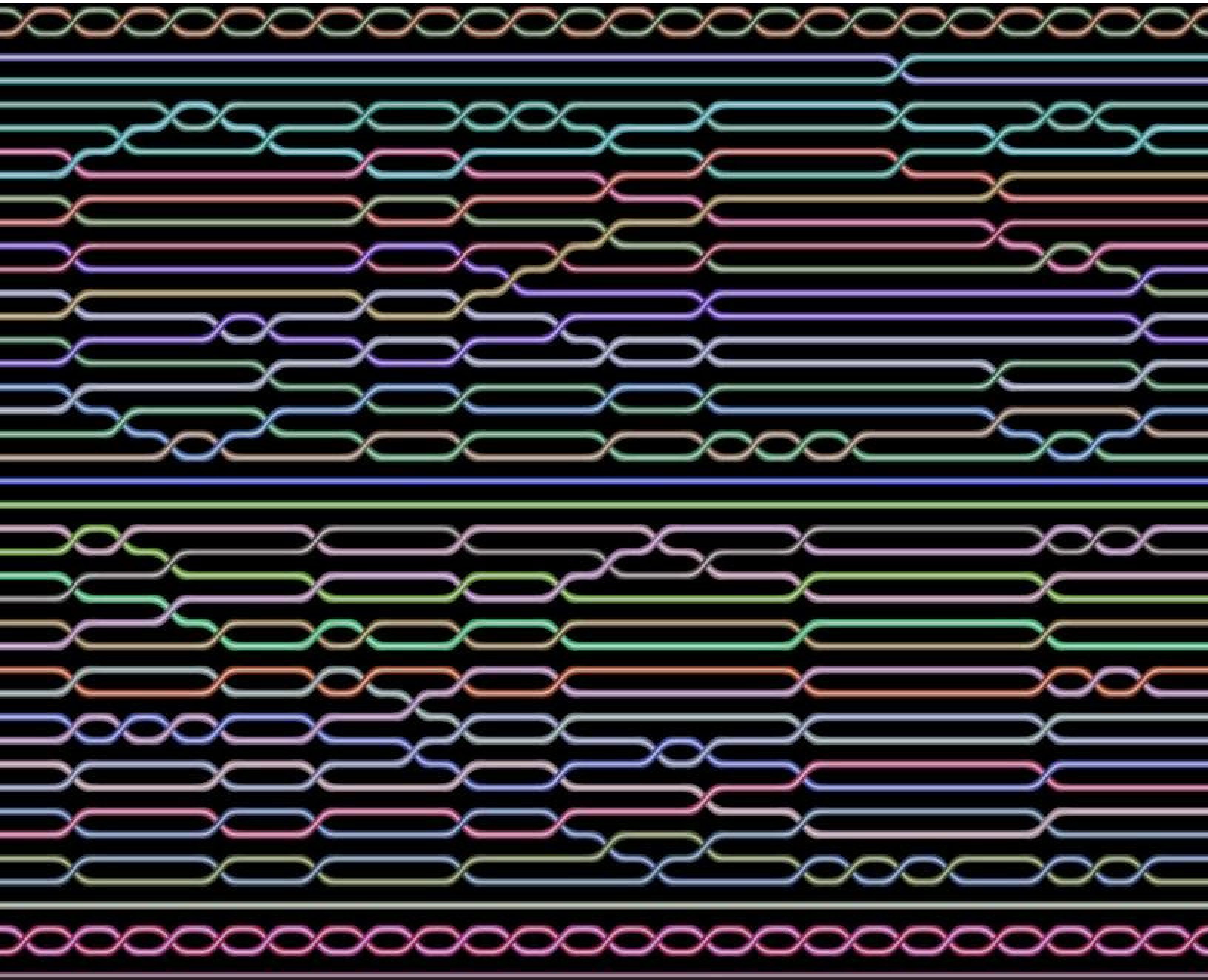
To Foster and Nurture Girls' Interest in Mathematics



From the Founder

Through the years, I've witnessed many people problem-solving. Perhaps the biggest impediment is dismissing thoughts, even one's own, without fair consideration, when they could actually unlock a solution. So, please stop dismissing thoughts! -Ken Fan, President and Founder

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Girls' Angle Bulletin

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Girls' Angle welcomes submissions that pertain to mathematics.

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Editors: Amanda Galtman
Jennifer Sidney
Executive Editor: C. Kenneth Fan

Girls' Angle:
A Math Club for Girls

The mission of Girls' Angle is to foster and nurture girls' interest in mathematics and empower them to tackle any field no matter the level of mathematical sophistication.

On the cover: *Girls' Angle Braided* by C. Kenneth Fan. For more on braids, see Hanna Mularczyk's Summer Fun problem set "A Salon in Nonabelia" on page 18.

An Interview with Rachel Roe-Dale

Rachel Roe-Dale is Professor of Mathematics and Statistics as well as the Program Director of the First-Year Experience at Skidmore College. She obtained her Doctor of Philosophy in Mathematics from Rensselaer Polytechnic Institute and a Bachelor's degree in chemistry from Maryville College. Her doctoral dissertation is entitled "Quantitative Models in Cancer Chemotherapy."

As Program Director of the First-Year Experience, she works to ensure that first-year students at Skidmore have a rewarding experience.

This interview was conducted by Naomi Danison and Ken Fan.

Ken: Thank you so much for agreeing to be interviewed by Girls' Angle! On your website, we saw that you majored in chemistry in college. When and how did mathematics enter into your life? Were you always interested in math, or did your interest come after college?

Rachel: You're welcome—thanks for reaching out to me! I have always enjoyed math, but I was not really exposed to the field of applied math until late in my undergraduate education when I studied differential equations. Then a few years later I started a graduate program in engineering at Rensselaer Polytechnic Institute and took a course called *Math Problems in Biology and Medicine*. This course showed me how incredibly powerful mathematics was as a tool and enabled me to look at many of the same problems in chemistry, biology, and medicine that I was interested in, but to use math, simulation, and computer programming to explore these questions rather than doing so in the lab with experiments. I switched my Ph.D. degree program to mathematics later that term.

…take a course in an area or field that you have never had the chance to explore before. Even for mathematics students, these connections will come up in surprising ways!

Ken: Your PhD thesis was about quantitative models of cancer chemotherapy. How did you apply mathematics to cancer chemotherapy?

Rachel: Beginning with the course I mentioned earlier, I began to explore how math modeling could be used to simulate and describe drug sequencing for cancer chemotherapy. For example, there is a simple case we can use to show how the order of treatment is important when treating with 2 drugs: drug A and C given twice each but in a different order can produce a quantitatively different outcome. The number of surviving cells depends upon whether we treat according to the sequence AACC or ACAC. Mathematically, the key result is to show that matrix multiplication is not commutative. Biologically and medically, that result can be explained by the knowledge that drugs affect cells differently depending upon which stage of the cell cycle they are in. The mathematical models that we formulated took these biological properties into account. More broadly speaking the role of mathematical modeling is to explore or offer explanations as to why observed phenomena is occurring and to hopefully guide future experimental studies.

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So, please consider subscribing to the Bulletin. Thanks to our sponsors, subscriptions cost $36/year. With a subscription, you have also gained access to our mentors via email and the ability to influence content in this Bulletin. Visit www.girlsangle.org/page/bulletin_sponsor.html for more information.

Thank you and best wishes,
Ken Fan
President and Founder
Girls' Angle: A Math Club for Girls

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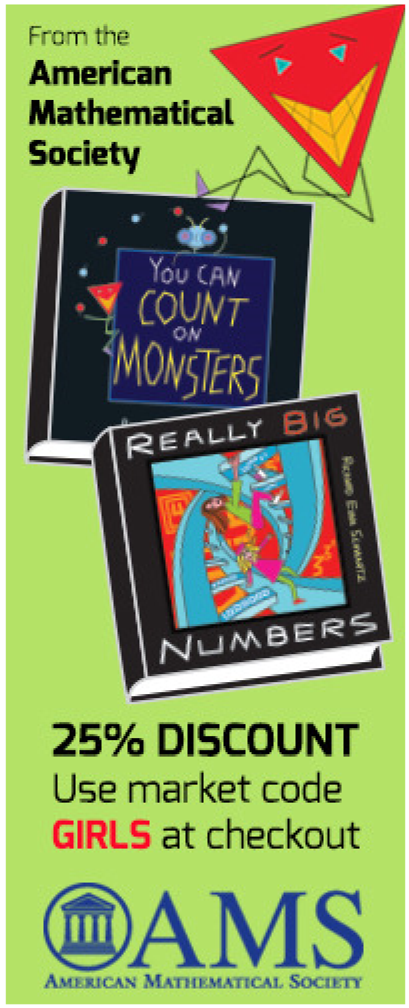


The American Mathematical Society is generously offering a 25% discount on the two book set *Really Big Numbers* and *You Can Count On Monsters* to readers of this Bulletin. To redeem, go to http://www.ams.org/bookstore-getitem/item=MBK-84-90 and use the code "GIRLS" at checkout

Ken: For students interested in applying mathematics to medicine, what are the most important math topics that they should study?

Rachel: Applied mathematics courses are helpful when considering medical applications. For example, differential equations, dynamical systems, numerical algorithms, and other modeling courses have interesting applications to medicine. Writing and oral communication courses are crucial because it is important to be able to communicate your results. In the same way, statistics, data visualization, and even art or art history courses are important because they are useful for gaining visual literacy and understanding how our vantage and point of view affect how we see, what we see, and how we relate to others. These courses can help students interpret and communicate visual data and information.

Ken: Do you feel that US K12 education does a good job of preparing students to be math majors in college? How can it be improved?

Rachel: Over the two decades I have been at Skidmore, I have found our first-year students enter with more and more mathematics background. For some students this advanced study during secondary school has prepared them to quickly jump into higher levels of mathematics courses which eventually allows them to take a wider range of courses before they graduate. But for other students, while they may have studied calculus, their foundational skills are still a bit rusty. I would much rather students enter with a solid foundational background in pre-calculus, trigonometry, etc. and wait until college to take Calculus and other higher-level courses. For that reason, I sometimes advise students to retake Calculus during their first term of college; this solid foundation is so crucial for future success.

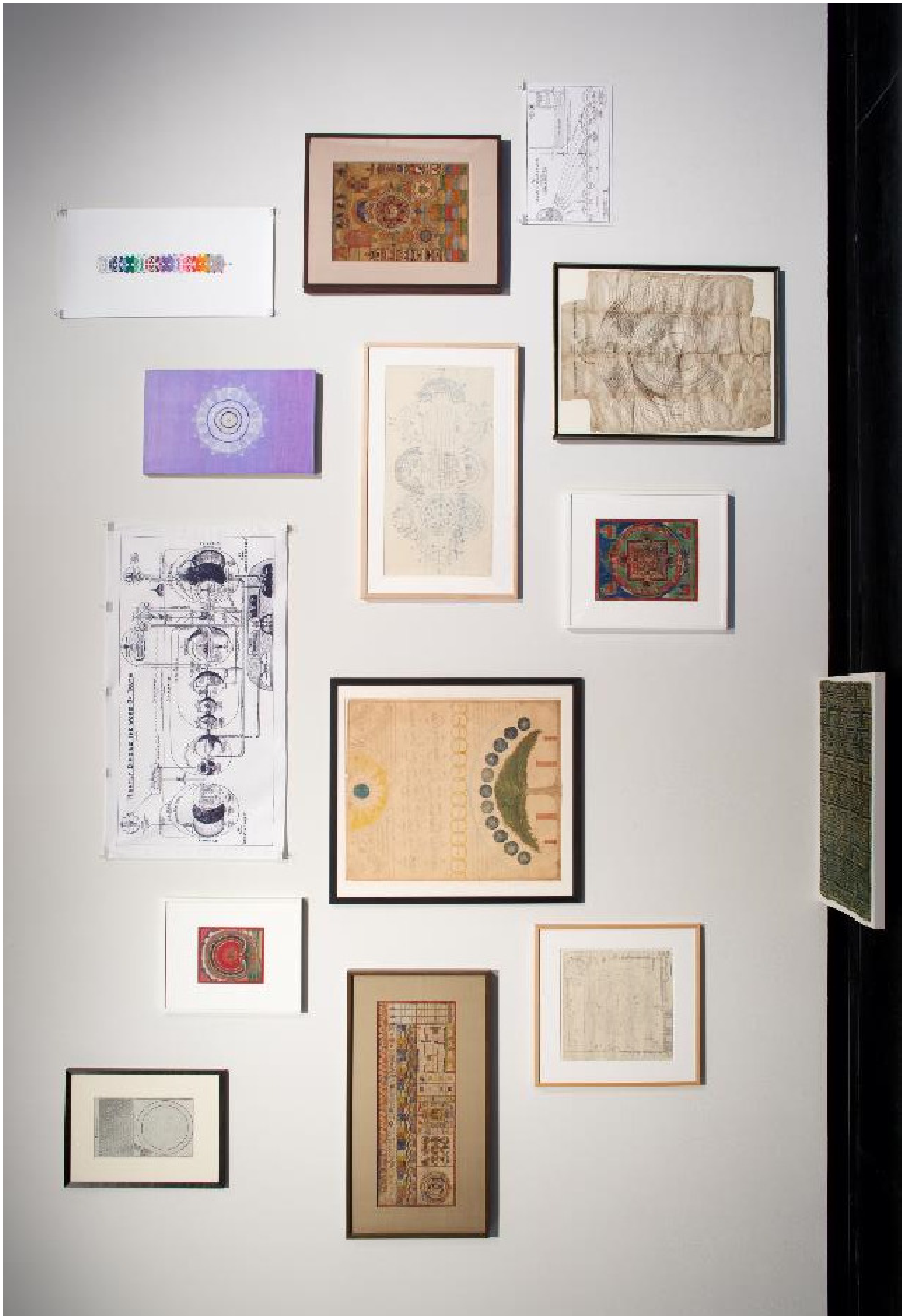
Ken: Do you believe there is gender bias in the field of mathematics? If so, what should be done to eradicate it?

Rachel: The mathematics community has made a lot of progress to be open and accommodating to individuals from all backgrounds. However, women and minorities are still underrepresented in Ph.D. programs and in academic positions in the US so there is certainly more room to improve and eradicate bias of all types. We need to make a commitment to support and promote inclusion of all individuals at all levels of mathematics study—from elementary school through higher education. Some examples of specific actions taken by my department at Skidmore are that we have had training to support inclusive hiring, and we've created a book group to read and discuss DEI issues in our field and pedagogy. Our professional societies such as the Association for Women in Mathematics also help promote equal access and do so through programming, funding opportunities, and mentoring.

Ken: Thank you for this interview!

A central feature of Skidmore's First-Year Experience is their Summer Reading Program. Check out the wonderful books they have read through the years for this program. Last year's book was Kazuo Ishiguro's *Klara and the Sun*. This year's book is Suleika Jaouad's *Between Two Kingdoms*.

Facing page: Installation view, *Sixfold Symmetry: Pattern in Art and Science*, 2016, image courtesy of The Frances Young Tang Teaching Museum and Art Gallery at Skidmore College, photograph by Arthur Evans.



Permutations and basic group theory: Part 2[1]

by Robert Donley[2]
edited by Amanda Galtman

The previous installment started our introduction to **group theory**, the mathematical language for studying symmetries. For this part, we apply group theory as a language to study symmetries using **group actions**, with the orbit-stabilizer theorem as a main result. This installment provides many opportunities to practice with cycle notation. We maintain definitions and notations from Part 1, and topics to review from that part include the definition of a group, the Cancellation Law, order of both a group and a group element, the abelian property, permutations, and cycle notation for permutations.

Definition: Define $[n] = \{1, 2, 3, \ldots, n\}$. The symmetric group S_n is the set of permutations on $[n]$ with the multiplication given by composition. We denote the identity element by $\mathbf{1}$.

To implement composition, we join the cycles and apply cycles on elements from right to left.

Example: If $\sigma = (12345)$ and $\tau = (12)(345)$ in S_5, then $\sigma\tau = (12345)(12)(345)$ which simplifies to $\sigma\tau = (1354)$ in cycle notation. Note that $\sigma\tau$ fixes 2. Products of cycles always simplify to a product of cycles with no shared numbers, also called **disjoint cycles**.

Exercise: Prove that the inverse of a permutation in cycle notation is given by reversing the entries in each cycle. For instance, verify that $(1234)(4321) = \mathbf{1}$ and $(123)(456)(321)(654) = \mathbf{1}$.

We define permutations on any finite set X by labeling the elements $x_1, \ldots, x_n$ and applying permutations in S_n to the indices: for σ in S_n, we have $\sigma(x_1) = x_{\sigma(1)}$.

A nonempty subset H of a group G is called a **subgroup** of G if H is also a group with the same multiplication. The groups in our examples will be subgroups of some S_n.

Exercise: Prove that a nonempty subset H of a finite group G is a subgroup of G if H is closed under multiplication. That is, assuming that xy is in H for all x and y in H, prove the identity and inverse properties for H.

One way to produce many subgroups of S_n is to choose a mathematical structure, such as a finite graph, and form the set of all symmetries that preserve the underlying structure. These symmetries are described mathematically by **group actions**.

Definition: Let G be a finite group and X a finite set with n elements. A **group action** of G on X is a function F from G to S_n, the symmetric group on the elements of X, such that, for all g, h in G and x in X,

- $F(gh)(x) = F(g)(F(h)(x))$, and
- $F(\mathbf{1})(x) = (x)$.

[1] This 15th installment, in a series that begin in Volume 15, Number 3, is also an informal sequel to "Learn by Doing: A Game of Throws 1" in Volume 7, Number 2 of this Bulletin.
[2]This content is supported in part by a grant from MathWorks.

The first condition states that the action respects group multiplication; that is, when applying consecutive group elements to x, we get the same answer if we multiply these elements in the group first. The second condition ensures that the identity element acts as the identity function. When the meaning is clear, we omit the function notation; that is, we may write gx for $F(g)(x)$.

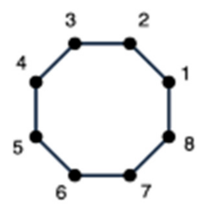
Example: By definition, the group S_n acts on the set $X = [n]$ as permutations.

Exercise: Prove that, for all g in G and x in X,

$$F(g^{-1})(x) = F(g)^{-1}(x) \text{ and } F(g^k)(x) = F(g)^k(x).$$

Note that the set of all $F(g)$ forms a subgroup of S_X, the group of all permutations on X.

Group actions let us visualize the group G as a set of symmetries that permute the elements of X. The following example and exercises generalize the symmetry groups for the triangle and square from the previous installment.



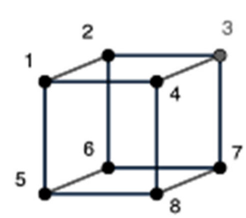
Example (Dihedral groups): The octagon at right has 16 symmetries that preserve its shape; call the group of these symmetries D_{16}. We count these symmetries with the Matching Rule in two steps: we can send the vertex 1 to any of the eight vertices and, independent of this choice, orient the adjacent edges in two ways. Thus, there are $2 \times 8 = 16$ symmetries of the octagon. If we include the identity with the rotations, the orientation splits these symmetries into eight rotations and eight reflections. We can describe these symmetries using a group action by mapping D_{16} to S_X, where X is the set of vertices. For instance, the rotation counterclockwise by 45° is $R \equiv (12345678)$ in cycle notation. There are two types of reflections. The reflection across the horizontal axis is $C \equiv (18)(27)(36)(45)$; no vertex is fixed under this reflection. On the other hand, the reflection across the axis through vertices 1 and 5 is given by $RC = (28)(37)(46)$, which has two fixed points.

Exercise: List the elements of D_{16} in cycle notation. Verify that $R^8 = C^2 = \mathbf{1}$ and $CRC = R^{-1}$. Prove that every rotation is of the form R^k for some non-negative integer k and every reflection is of the form CR^k. Prove that the product of two reflections is a rotation, that the product of a rotation and a reflection is a reflection, and that every rotation is the product of two reflections.

In the last exercise, we saw that all elements of D_{16} are products of R and C. We say that D_{16} is **generated** by R and C. In general, a group G is generated by a subset of its elements if and only if every element of the group can be obtained by multiplying elements of the subset.

Exercise: The dihedral group D_{2n} is the set of symmetries of a regular n-sided polygon. For uniformity, label the vertices consecutively 1 through n in counterclockwise order. Rework the previous example and exercise for this general case. What differences appear between the cases for even and odd n?

Exercise: Prove that there are 48 symmetries of the cube, including reflections. List each symmetry in cycle notation, find its order, and describe the symmetry geometrically.



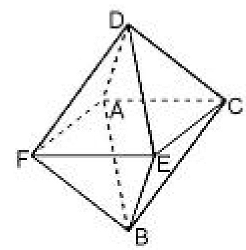
Exercise: Consider the symmetry group of the cube consisting of only rotations. Prove that there are 24 such rotations. List these rotations in cycle notation. Compare orders of elements and cycle structures.

Now, consider the tetrahedron; its symmetry group is equivalent to S_4. The cube contains two tetrahedra: one with vertices 1, 3, 6, and 8, and the other with vertices 2, 4, 5, and 7.

Exercise: Which symmetries of the cube preserve each tetrahedron, and which symmetries interchange them? Prove that every symmetry of the tetrahedron with vertices 1, 3, 6, and 8 induces (i.e., extends uniquely to) a symmetry of the cube.

Exercise: Prove that every symmetry of the cube is a product of a rotation and the reflection $C \equiv (12)(34)(56)(78)$.

Exercise: Consider the subgroup H with eight elements generated by the reflections $(12)(34)(56)(78)$, $(14)(23)(58)(67)$, and $(15)(26)(37)(48)$. Prove that H is abelian and that every element of H has order 2.

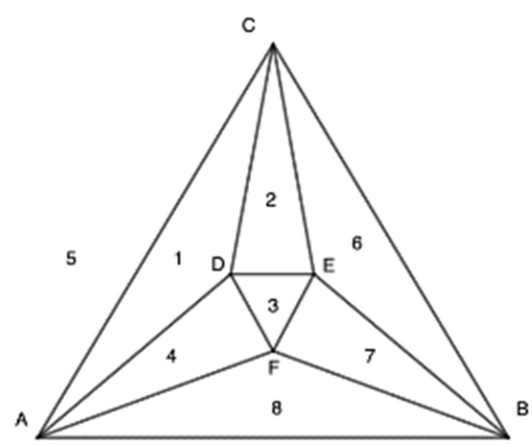


Let's consider one more Platonic solid, the octahedron, as pictured on the right.

Exercise: List the symmetries of the octagon in cycle notation using letters for vertices. How many rotations are there?

Fit the cube into the octagon by placing each vertex of the cube at the center of a face of the octagon. One possible placement is denoted by the flattened octagon below; face ABC of the octagon corresponds to vertex 5 of the cube.

Exercise: In your listing of the symmetries of the octagon using vertices, also represent each symmetry as a permutation of the faces using numbers. For each element, verify that the orders of both permutations agree.



The previous exercise highlights a common situation when working with groups. The symmetry group for the octagon is represented by vertex symmetries in S_6 and by face symmetries in S_8. If we were given these subgroups of S_6 and S_8 directly, it might not be clear that they represent the same group of symmetries!

Exercise: Count the rotational symmetries of the remaining Platonic solids: the dodecahedron and the icosahedron. Count the numbers of vertices, faces, and edges for each solid, and create a relationship like the one between the cube and octagon.

Exercise: Explore the symmetry properties of the Archimedean solids.

Further group theoretic properties and counting methods are obtained through the **orbits** of a group action.

Definition: Fix x in X. The set $O_x = \{gx \mid g \text{ in } G\}$ is called the **orbit** of x under G (or the G-orbit of x).

The orbits partition X into disjoint subsets. For elements in the same orbit, there is a symmetry that maps one to the other, whereas no symmetry maps an element to a different orbit.

Definition: If $X = O_x$ for some x in X, we say that the group action is **transitive**.

Example: For regular n-sided polygons and the Platonic solids, the group action is transitive on the vertices, edges, and faces. For the octagon, if we use instead the subgroup generated by R^2, then the vertices split into two orbits $O_1 = (1, 3, 5, 7\}$ and $O_2 = \{2, 4, 6, 8\}$.

Our main result is the orbit-stabilizer theorem. With this theorem, we will prove the multinomial coefficient formula with group theory and count the number of permutations having a given cycle structure.

Recall that if X is a finite set then $|X|$ is the number of elements in X. For instance, $|S_n| = n!$.

Definition: Fix x in X. The **stabilizer subgroup** of x under the action of G, denoted by $\text{Stab}_G(x)$, is the subset of all g in G such that $gx = x$.

Exercise: Prove that $\text{Stab}_G(x)$ is a subgroup of G.

Elements in the stabilizer of x have no effect on x and pertain to the overcount in the theorem below.

Orbit-stabilizer theorem: Let G be a finite group acting on a finite set X. Fix x in X, and let O_x be the G-orbit of x. Then $|O_x| = |G|/|\text{Stab}_G(x)|$.

If G acts transitively on X, then $|X| = |G|/|\text{Stab}_G(x)|$ for any x in X.

Exercise: Before reading the proof below, try to prove the orbit-stabilizer theorem.

Proof of the orbit-stabilizer theorem: By restricting to O_x, where G acts transitively, we see that we may assume, without loss of generality, that G acts transitively on X. Fix x in X. Suppose $y = gx$ for some g in G. If h is in $\text{Stab}_G(x)$, then $ghx = gx = y$. The Cancellation Law implies that the elements gh are distinct as h ranges over the elements of $\text{Stab}_G(x)$. On the other hand, if $y = gx = kx$, then $g^{-1}kx = x$, so $g^{-1}k$ is in $\text{Stab}_G(x)$, so $k = gh$ for some h in $\text{Stab}_G(x)$. In other words, the map from G to X which sends g to gx is onto and the preimage of any element of X has exactly $|\text{Stab}_G(x)|$ elements. Therefore, $|X| = |G|/|\text{Stab}_G(x)|$. □

Example: We use the theorem to count the number of compositions associated with a partition. For the partition 4321 of 10 with four parts, there are 4! = 24 ways to rearrange the parts to obtain a similar composition. In this case, group elements act by permuting the positions in the list, so $G = S_4$ and $\text{Stab}_G(4321) = \mathbf{1}$. Thus $|O_{4321}| = 24/1 = 24$, as expected.

Example: Consider the partition 5221. Then $\text{Stab}_G(5221) = \{\mathbf{1}, (23)\}$ and $|O_{5221}| = 12$.

Exercise: List the 12 compositions associated with the partition 5221.

Example: Consider the partition 333221 of 14 with six parts. Now $G = S_6$ and $\mathrm{Stab}_G(333221)$ has 12 elements. So $|O_{333221}| = 720/12 = 60$.

Exercise: Use the Matching Rule to prove that 333221 is stabilized by 12 elements. List these elements in cycle notation. Then list all 60 compositions in the orbit by first listing all 10 compositions associated with 33322. Consider the six ways to insert the 1 and use the Matching Rule again to count all 60 elements in the orbit.

Example (Multinomial coefficients): Consider a partition λ in which the part λ_i occurs a_i times. If $k = a_1 + \ldots + a_n$ is the number of parts, then $G = S_k$ and, by the Matching Rule, there are $a_1!\cdots a_n!$ permutations that fix λ. Thus, there are $C(a_1, \ldots, a_n) = k!/(a_1!\cdots a_n!)$ compositions corresponding to λ.

Exercise: Rework the multinomial coefficient examples from the previous installment using group actions and stabilizers. Determine the orbits under S_k, and verify the orbit-stabilizer theorem. Each orbit should contain a unique partition, possibly filled out with zeros.

Next, we revisit the exercise about counting the elements in S_n with a given cycle structure, from the end of the previous installment. A permutation consisting of a single cycle such as (1234) is unique up to the cyclic ordering of elements in the cycle: (1234) = (2341) = (3412) = (4123), but (1234) ≠ (1324).

In general, the number of cycles of length k is $P(n, k)/k$, where $P(n, k) = n!/(n - k)!$ is the number of k-permutations on n elements. For general permutations with different cycle lengths, we use the Matching Rule to compute the size of stabilizers. If a cycle length occurs with multiplicity m, there are $m!$ ways to order these cycles independently of the cyclic orderings in each cycle.

Example: In S_5, the possible cycle structures for nonidentity elements are

$$(ab),\ (abc),\ (abcd),\ (abcde),\ (ab)(cd),\ (ab)(cde).$$

The number of elements of each type are 10, 20, 30, 24, 15, and 20, respectively. As a check, the sum of these numbers plus 1 is 5! = 120.

Exercise: Verify these counts.

Exercise: Repeat the previous example for S_1, S_2, S_3, S_4, and S_6.

We consider a generalization of the permutation counting problem by cycle structure in S_n. In this case, the group plays the role of both G and X in the group action.

Definition: Let G be a finite group. Define the action of **conjugation** of G on itself by

$$C(g)x = gxg^{-1}.$$

We call gxg^{-1} the **conjugate** of x by g. The orbit of x under the conjugation action is called the **conjugacy class** of x.

Exercise: Verify that conjugation is a group action. Describe $\mathrm{Stab}_G(x)$ for the conjugation action. Describe the conjugacy classes of G when G is abelian.

Exercise: The **center** of G, denoted by $Z(G)$, consists of all elements that commute with all elements of G; that is, g is in $Z(G)$ if and only if $gh = hg$ for all h in G. Prove that $Z(G)$ is an abelian subgroup of G and describe the center in terms of the conjugacy classes of G.

Example (dihedral groups): Consider a regular polygon with n vertices. The dihedral groups D_{2n} split into cases depending on the parity of n. Define R and C similarly to what we did for dihedral groups earlier. Suppose n is odd (so C has only one fixed vertex). For the conjugacy class of any rotation R^i, check that $(CR^k)R^i(CR^k)^{-1} = R^{-i}$. For rotations, the conjugacy classes are given by the identity class $\{\mathbf{1}\}$ and the $(n-1)/2$ pairs $\{R^i, R^{n-i}\}$, for $i = 1, 2, 3, \ldots, (n-1)/2$. The n reflections form their own class $\{CR^i\}$ since $R^{-i}CR^i = CR^{2i}$. The conjugacy classes partition the group, giving a sum formula known as the **class equation**: $2n = 1 + (2 + \ldots + 2) + n$.

Exercise: For odd n, prove that every reflection CR^k with $1 < k < n-1$ is of the form CR^{2i}.

Exercise: List the elements of D_{10} in cycle notation. Note that elements in the same conjugacy class have the same cycle structure.

Exercise: Adapt the conjugacy class description for D_{2n} in the even case. The subset of reflections splits into two classes, as seen for D_{16}. Describe the reflection classes in cycle notation and explain the difference geometrically. Also, describe the center in cycle notation and explain it geometrically.

To finish, conjugation provides a way to both count and list all permutations of a given cycle structure. We omit the proof of the following theorem, which shows that the conjugacy classes in S_n are determined by cycle structure.

Theorem (conjugacy in the symmetric group): If σ and τ are in S_n, then $\tau\sigma\tau^{-1}$ has the same cycle structure as σ. To obtain the cycle entries of $\tau\sigma\tau^{-1}$, we replace each number m in a cycle representation for σ with $\tau(m)$.

For instance, (123) sends 1 to 2 and 2 to 3, and, indeed, $(123)(12)(123)^{-1} = (23)$.

Exercise: Verify that $(1234)(3452)(1234)^{-1} = (4153)$ using composition of cycles.

Exercise: Determine the 18 elements in the stabilizer of (123)(456) in S_6.

Exercise: Write down all the elements of S_3 and S_4 and group them into conjugacy classes. Give the class equation as in the dihedral group example. Are there any nonidentity elements in the center of these groups? Are there nonidentity elements in the center of any S_n?

Exercise: Repeat the previous exercise for D_{10}, D_{16}, and the symmetry group of the cube, with elements represented in cycle notation.

Exercise: Prove the theorem about conjugacy in the symmetric group.

Cubics, Part 3

by Lightning Factorial | edited by Jennifer Sidney

In the spirit of figuring things out, we asked Lightning Factorial to try to find a formula for the roots of a cubic equation in terms of its coefficients. The cubic formula, like the quadratic formula, is well known and can readily be looked up. But trying to figure out something yourself can take you on a journey that's far more fun. Let's rejoin Lightning's cubic math adventure!

There's a lot to sort out!

I think I'll carefully go through the steps I found last time to try to bring clarity to some of the things I was confused about. I was especially concerned that the method seemed to yield six solutions instead of just three.

Everything seemed fairly clear up to the point where the problem was reduced to solving cubic equations of the form $x^3 + cx + d = 0$.

Then, I decided to substitute $u + v$ for x in the hope that solutions to such cubic equations could be written as a sum of two quantities, u and v, each of which can be found more easily than solving the original cubic equation. After some algebraic simplification, I arrived at the equation

$$u^3 + (3uv + c)(u + v) + v^3 + d = 0.$$

By insisting that $3uv + c = 0$, this equation becomes a quadratic equation in $U = u^3$:

$$U^2 + dU - c^3/27 = 0. \qquad (*)$$

This quadratic equation will generally have two roots, which I'll call U_+ and U_-.

To recover u, I take a cube root of a solution to the quadratic equation (*). In general, U_+ will have three cube roots and U_- will also have three cube roots. I'll let u_{+1}, u_{+2}, and u_{+3} be the three cube roots of U_+ and let u_{-1}, u_{-2}, and u_{-3} be the three cube roots of U_-.

Then, I let $v = -c/(3u)$, where u is a cube root of one of the roots of the quadratic equation (*). So I'll define $v_{+k} = -c/(3u_{+k})$ and $v_{-k} = -c/(3u_{-k})$, for each $k = 1, 2, 3$.

Then $u_{+k} + v_{+k}$ and $u_{-k} + v_{-k}$ are solutions to the cubic equation $x^3 + cx + d = 0$, for each $k = 1, 2, 3$.

But there should be at most three solutions to the cubic, not six! It must be that some of the expressions $u_{+k} + v_{+k}$ and $u_{-k} + v_{-k}$ are equal to each other.

Actually, there's a symmetry between u and v, since all I was hoping for was to find solutions of the form $u + v$… but $u + v = v + u$. I see. I think that explains it! The v_+'s must correspond to the u_-'s. Indeed, the equation $v_{+k} = -c/(3u_{+k})$ can be rewritten

$$v_{+k}u_{+k} = -c/3.$$

If we cube both sides, we get

$$(v_{+k})^3(u_{+k})^3 = -c^3/27.$$

And we know that $(u_{+k})^3 = U_+$. By Vieta's formulas, we know that the product of the roots of the quadratic equation (*) must be equal to its constant term (divided by its leading coefficient), which is $-c^3/27$. Therefore v_{+k} is, in fact, a cube root of U_-.

The upshot is that I really only need to consider one of the roots of the quadratic equation (*); I can take $u_{+k} + v_{+k}$, for $k = 1, 2, 3$, to be the three solutions to the cubic. (Alternatively, the three roots of the cubic are also equal to $u_{-k} + v_{-k}$ for $k = 1, 2, 3$.)

Since I need to divide by u_{+k} in order to compute v_{+k}, I can't have u_{+k} equal to zero. So I should examine what happens in the case where u_{+k} *is* zero.

If $u_{+k} = 0$, then the quadratic equation (*), which is $U^2 + dU - c^3/27 = 0$, must have a constant term equal to zero. So if $u_{+k} = 0$, then $c = 0$ and the cubic equation becomes $x^3 + d = 0$. Fortunately, this cubic can readily be solved. Its solutions are the cube roots of $-d$.

Great! My confusions are cleared up!

So here are the steps I've found to solve a cubic equation:

1. First, divide the cubic throughout by its lead coefficient to obtain a cubic of the form $x^3 + bx$ + (some constant)x + (another constant). This does not change its roots.

2. Translate the cubic horizontally to the left by $-b/3$ to obtain a cubic of the form $x^3 + cx + d$. This translates the root accordingly.

3. If $c = 0$, then the roots of $x^3 + cx + d$ are the cube roots of $-d$, and we're essentially done. All we have to do is translate the cube roots of $-d$ to the right by $-b/3$.

4. If $c \neq 0$, then let U be a root of the quadratic equation $U^2 + dU - c^3/27 = 0$. It doesn't matter which root we take.

5. Let u_1, u_2, and u_3 be the cube roots of U.

6. Let $v_k = -c/(3u_k)$.

7. Then the roots of $x^3 + cx + d$ are $u_k + v_k$, for $k = 1, 2, 3$.

8. The roots of the original cubic are $u_k + v_k - b/3$, for $k = 1, 2, 3$.

Let's test this procedure!

To test it, I'll use a cubic whose roots I already know. I'll test it on $(x - 1)(x - 2)(x - 3)$, which expands to $x^3 - 6x^2 + 11x - 6$. The procedure had better yield the roots 1, 2, and 3! Let's see!

For step one, the lead coefficient is already 1, so I move on to step two and translate to the left by 2. That is, I make the substitution $x = X + 2$. After some algebra, I get the cubic $X^3 - X$.

Even though I can see how to factor $X^3 - X$ (and, indeed, I know that the roots are -1, 0, and 1), I'm doing this to test that the procedure works. So, continuing with the procedure, I see that the coefficient of the linear term is *not* equal to 0, so I must find a root of the quadratic equation $U^2 + 1/27 = 0$. One of the roots of this quadratic is $i/(3\sqrt{3})$, where i is the square root of -1.

The three cube roots of $i/(3\sqrt{3})$ are $w/\sqrt{3}$, $w^5/\sqrt{3}$, and $w^9/\sqrt{3}$, where $w = \dfrac{\sqrt{3}}{2} + \dfrac{1}{2}i$ is one of the primitive twelfth roots of unity. For each such cube root u, I am supposed to compute $1/(3u)$. I get $w^{11}/\sqrt{3}$, $w^7/\sqrt{3}$, and $w^3/\sqrt{3}$, respectively.

Gosh, these are strange complex numbers! Is this really going to work?

According to the procedure, the roots of $X^3 - X$ should be

$$w/\sqrt{3} + w^{11}/\sqrt{3},\ w^5/\sqrt{3} + w^7/\sqrt{3},\ \text{and}\ w^9/\sqrt{3} + w^3/\sqrt{3}.$$

Hey, it does work! Because $w + w^{11} = \sqrt{3}$, $w^5 + w^7 = -\sqrt{3}$, and $w^9 + w^3 = 0$, these three quantities are equal to 1, -1, and 0, which are the roots of $X^3 - X$!

Translating to the right by 2, that is, adding 2 to each of these roots, yields 3, 1, and 2, which are, indeed, the roots of our test cubic.

How strange that the procedure travels through the complex plane before yielding real solutions!

What's going on here?

I guess I will try to understand when a cubic equation has three real roots (counted with multiplicity). Since the first two steps of the procedure alter the roots by translation, I'll focus my attention on cubics of the form $x^3 + cx + d$.

First of all, if $x^3 + cx + d$ has real roots, must c and d be real? Vieta's formulas show that this is indeed the case, for c is the sum of the products of the roots in pairs and d is the negative of the product of the roots.

So my next question is: For what real values of c and d does $x^3 + cx + d$ have three real roots?

Since x^3 and x both strictly increase with x, if $c > 0$, the cubic $x^3 + cx + d$ will be strictly increasing, thus cannot have three real roots. And if $c = 0$, the cubic $x^3 + d$ has three real roots only when $d = 0$, when 0 is a triple root.

But what about $c < 0$?

Can you figure out for which values of c and d the cubic $x^3 + cx + d$ has real roots? For more questions on cubic equations, check out the Summer Fun problem set "Cubics" on page 26.

Summer Fun!

The best way to learn math is to do math. Here are the 2024 Summer Fun problem sets.

We invite all members and subscribers to send any questions and solutions to us at girlsangle@gmail.com. We'll give you feedback and might put your solutions in the Bulletin!

The goal may be the lake, but who knows what wonders you'll discover along the way?

In the August/September issue, we will provide some solutions. You could wait to see the answers, but you will learn a lot more if you try to solve these problems on your own first.

Some problems are very challenging and could take several weeks to solve, so please don't approach these problems with the idea that you must solve them all. Our main goal is to give you some interesting things to think about.

If you get stuck, try to formulate a related question that you can see a way to actively explore to get your mind moving and your pencil writing. If you don't understand a question, email us.

If you're used to solving problems fast, it can be frustrating to work on problems that take weeks to solve. Try to enjoy the journey and don't be afraid to follow detours. It's like hiking up a mountain. Getting to the top rewards one with a spectacular view, but during the journey, there's a lot to see and experience. So here's a meta-problem for those of you who feel frustrated when doing these problems: see if you can dissolve that frustration and replace it with a relaxed, optimistic sense of adventure!

This is Summer Fun, not Summer Torture!

Summer Fun!

A Salon in Nonabelia

by Hanna Mularczyk

This summer break, instead of a traditional summer camp, your parents decide it’s time for you to learn more about the world, so they ship you off, alone, to the far and distant land of Nonabelia.

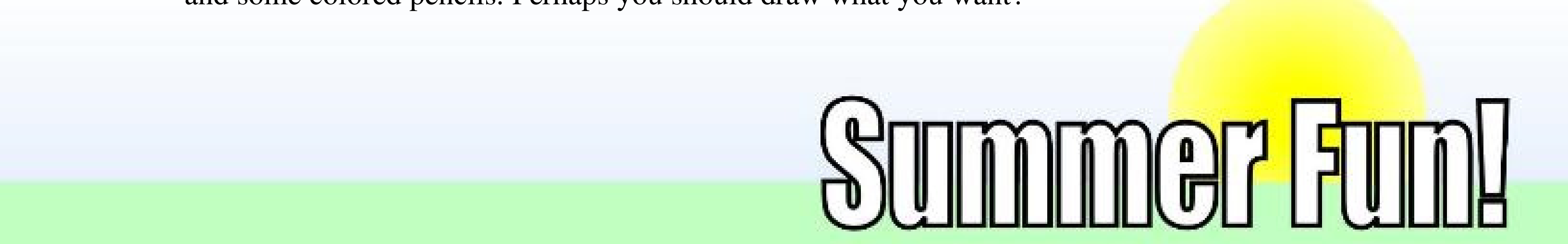
What weather.com failed to tell you is that Nonabelia is terribly windy in the summer, and your long, heavy hair keeps blowing in front of your face, making it difficult to enjoy or even navigate the arid planes, plateaus, and tunnels of the land.

Luckily, shortly after you arrive, you pass upon a village that, from the photos pasted in the window, seems to have a hair salon.

Perfect! You walk in and decide you don’t want to cut off all of your hair, and a simple ponytail won’t do–you’ll need a hairstyle that will keep your hair back tightly and can stay in a while, so you sit down at an open chair and say to the stylist, “I’d like a braid, please.”

She stares back at you blankly.

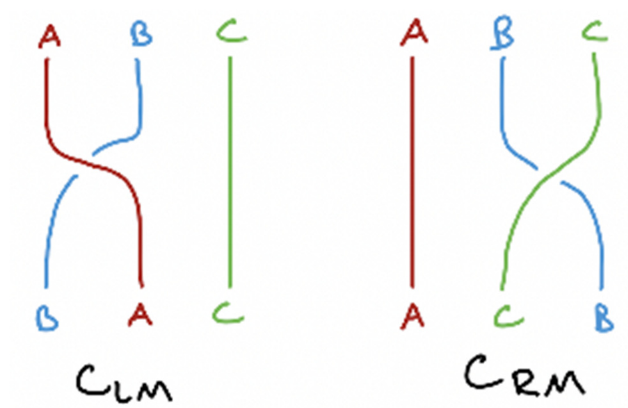
It dawns on you that you haven’t heard anyone in Nonabelia speak a word of English, or any language for that matter. Though she seems to understand the confusion and hands you a notepad and some colored pencils. Perhaps you should draw what you want?



Alright. How hard can this be? You remember that earlier this year you dyed your hair in three streaks of red, blue, and green. How convenient! You draw on the pad a red line, a blue line, and a green line, and label them A, B, and C, respectively. The stylist seems to understand, and she separates your hair into the three strands, holding them at the bottom with her hands.

You make a mental note that while strands A, B, and C are initially in the left, middle, and right positions on your head, respectively, once you start braiding they might end up in different positions. So "strand A" will always mean the strand that started on the left, shown in red, whereas "left strand" will mean the strand that is currently in the left position (at the bottom of the diagram).

You figure it's best to start with the classic three strand braid. It starts with moving the left strand over the middle strand. You want to keep track of which strand went over which, so you draw the left strand on top of the middle strand at their crossing. The second move is similar, now with the right crossing over the middle strand. So you don't have to keep drawing them out, you label the first move c_{LM} (for "left over middle cross") and the second move c_{RM} (for "right over middle cross").



To do a sequence of moves, you can write them left to right in the order you want the stylist to do them (from top to bottom on your strands). So to get the braid you want, you show the stylist these pictures, along with $c_{\mathrm{LM}}c_{\mathrm{RM}}c_{\mathrm{LM}}c_{\mathrm{RM}}c_{\mathrm{LM}}c_{\mathrm{RM}}c_{\mathrm{LM}}c_{\mathrm{RM}}$, to tell her to do those moves in that order. And surely enough…you get your braid!

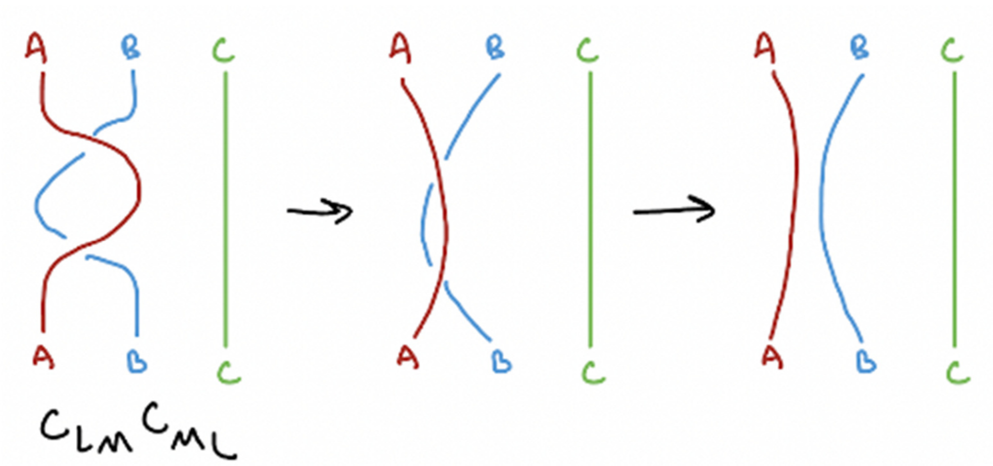
Summer Fun!

$c_{LM}c_{RM}c_{LM}c_{RM}c_{LM}c_{RM}c_{LM}c_{RM}$

But why stop there? With these two moves, you can make a lot of other, more interesting styles.

First, you'll have to undo the braid you just did to get back to the unbraided strands, which you will call the **unbraid**. To do this, you should undo each individual move, in reverse order.

For c_{LM}, you crossed the left strand over the middle strand. So to undo it, it would make sense to cross the middle strand over the left strand. Call this c_{ML}.



Here's what $c_{LM}c_{ML}$ looks like. Notice that it doesn't quite look like the unbraid yet. To get it to the unbraid, however, all you have to do is deform your drawing a bit, pulling the middle of strand A to the left and the middle of strand B to the right. So you want to consider two braids equivalent if the stylist can get from one to the other by keeping the ends of each strand fixed and moving around the stands a bit in 3-dimensional space (this would be great to practice at home with 3 strings, shoelaces, ribbons, or your friend's head).

Then $c_{LM}c_{ML} = c_{ML}c_{LM}$ = unbraid, so c_{LM} and c_{ML} are **inverses**.

Summer Fun!

Note that $c_{LM}c_{LM}$ is **not** the unbraid. If the stylist tries to pull the middle of strand A to the left, it cannot pass through strand B, since it now crosses under it.

1. What is the inverse of c_{RM}? Draw it, give it a name to match the other names, and show with a drawing how doing it before or after c_{RM} creates the unbraid.

2. Write down the sequence of moves that undoes the entire braid $c_{LM}c_{RM}c_{LM}c_{RM}c_{LM}c_{RM}c_{LM}c_{RM}$.

You decide to call c_{LM}, c_{RM}, c_{ML}, and c_{MR} moves **3-moves** and any style that you can get using them a **3-braid**. What sort of styles can you get with a 3-braid?

Like you noticed before, the strands start in ABC order, but they don't have to end in that order. In the examples above you have braids that end in BAC order and ACB order. You call this the **end order**.

3. For every ordering (**permutation**) of ABC, can you find a 3-braid with that end order? The ones you haven't gotten yet are BCA, CAB, and CBA. If possible, draw a braid with that end order that uses the least possible number of moves (we call this the **length** of the permutation).

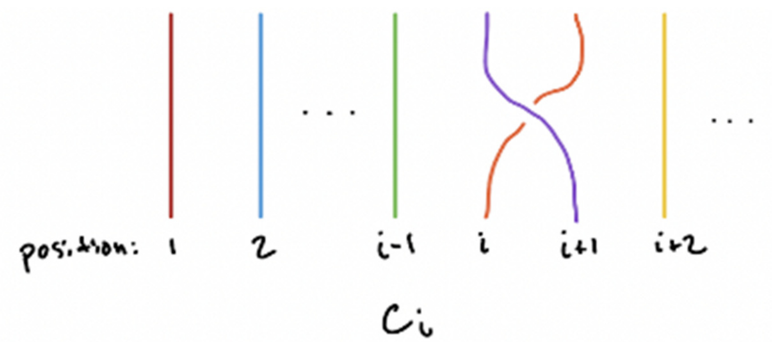
Even though you've been at the salon for a while now playing around with braids, the stylist shows no signs of fatigue and annoyance (in fact, she seems quite excited to see what you come up with next). You get to thinking: are there multiple different ways to get to the same ending 3-braid? As you already know, you could always add a $c_{LM}c_{ML}$ to your braid and that won't change it. But is there anything more interesting going on?

4. We say two moves **commute** if doing the first and then the second results in the same braid as doing the second and then the first. Which pairs of distinct moves commute? Which don't?

5. Show that $c_{LM}c_{MR}c_{LM} = c_{MR}c_{LM}c_{MR}$.

The stylist seems to notice you getting a little bored, and a thought hits her. She stands up and opens up a drawer to reveal…hair extensions!! A seemingly never-ending supply of them, in a seemingly never-ending number of colors. She can clip them onto your head in order to add new strands.

You decide to adapt your 3-braid language to support any number of strands. In general let c_i be the move that crosses the strand in the ith position from the left over the strand in the $(i + 1)$th position from the left, and c_i^{-1} be the move that crosses the $(i + 1)$th over the ith, so that c_i^{-1} is indeed the inverse of c_i (again, you remember these moves are based on the current position of the end of each strand, not where they started). An ***n*-braid** is any style you can get on n strands of hair by using c_1, c_2, …, c_{n-1} and their inverses, which you call the ***n*-moves**. You wonder if some of the previous styling you've done together generalizes. If you get stuck, try thinking about 4-braids first.



6. In general, given any permutation of the numbers 1, …, n, can you make an n-braid whose end order is that permutation? Why or why not?

7. Given c_i and c_j, when do they commute with each other, and when don't they?

8. What does the equation in problem 5 look like in this new setting? Can you generalize it?

9. Is there any n-braid (for any n) that you can repeat some number of times to get back to the unbraid? You don't need to prove it, but maybe think about why this might be the case.

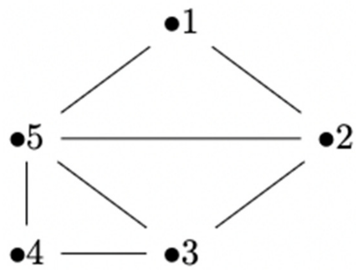
10. Design a braid on at least 4 strands that you think you'd like to wear in Nonabelia.

After generously tipping your stylist and promising to come back again, you leave the salon with your new, breathtakingly chic and mathematically intriguing braid. And, better yet, without your hair in your face you can finally see the gorgeous and bizarre new landscape that surrounds you. You smile and journey on.

Forbidden Subgraphs!

by Dora Woodruff

Before we start, there are a few definitions you should know. A **(simple) graph** is a collection of points, called **vertices**, and a collection of lines between the points, called **edges**. More formally, an edge is a pair of distinct vertices. In the example graph G below, the vertices of G are labeled 1, 2, 3, 4, and 5, and the edges of G are $\{1, 2\}$, $\{1, 5\}$, $\{2, 3\}$, $\{2, 5\}$, $\{3, 4\}$, $\{3, 5\}$, and $\{4, 5\}$.



The graph G

For most of this problem set, we'll assume that our graphs have a finite number of vertices. A **subgraph** of a graph is a subset of the vertices and edges in between those vertices–that is, another graph inside our original graph. An example of a subgraph of our graph G has vertices 1, 2, and 5, and edges $\{1, 5\}$ and $\{1, 2\}$. Another example of a subgraph: we could include all of the vertices 1, 2, 3, 4, and 5 and none of the edges.

1. Find some more subgraphs of G. How many subgraphs with two vertices are there?

A **(simple) cycle** is a list of vertices $v_1, v_2, v_3, \ldots, v_n$ such that $v_1 = v_n$, each v_i is connected to the next v_{i+1} in the list with an edge, and no other vertices in the list are equal to each other. For example, 1, 2, 5, 1 is a cycle in G, because $\{1, 2\}$ is an edge, $\{2, 5\}$ is an edge, and $\{1, 5\}$ is an edge. The length of the cycle is the number of distinct vertices in the cycle.

2. Find some more cycles in G. How long is its longest cycle?

The **complete graph** on n vertices, also called K_n, is the graph where all pairs of vertices are joined by an edge. For example, the complete graph K_2 on 2 vertices has just one edge, and the complete graph K_3 on 3 vertices is a triangle.

3. Draw pictures of the complete graph on 4 vertices and on 5 vertices. How many edges does each graph have? How many edges would the complete graph on 100 vertices have?

The complete graph on n vertices can also be thought of as the (simple) graph with n vertices that has the maximum possible number of edges, since every single pair of vertices contributes an edge.

Summer Fun!

In the rest of this problem set, we'll consider the general question: what is the maximum number of edges we can have if some patterns in our graph are forbidden?

Cycles are forbidden!

4. How many edges can you fit in a graph with 4 vertices without creating any cycles? What about 5 vertices? 6 vertices? Draw some examples of graphs that have a relatively high number of edges without having any cycles.

By the way, a graph that has no cycles is called a **tree**. Why do you suppose that is?

The **degree** of a vertex is defined as the number of edges coming out of it.

5. Show that if a graph has no cycles as subgraphs, then it must have a vertex of degree at most one. Assuming there is at least one edge, must the graph have at least two vertices of degree at most one?

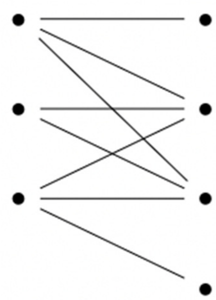
6. If we allow our graph to have an infinite number of vertices, is the claim of Problem 5 still true?

7. What is the maximum number of edges we can put in a graph with n vertices without creating any cycles?

Triangles are forbidden!

8. How many edges can you fit in a graph with 4, 5, or 6 vertices without creating any triangles as subgraphs? (A triangle is just a cycle with three vertices).

A **bipartite graph** is a graph which has 'two sides' in the following sense: there are some vertices on the left, some vertices on the right, and the only edges in the graph have one vertex on the left and one vertex on the right (there are no edges with both vertices on the same side). Here's an example, with 3 vertices on the left side and 4 vertices on the right:



9. Show that any cycle in a bipartite graph has an even number of vertices. In particular, this implies that bipartite graphs have no triangles.

10. If my bipartite graph has 2 vertices on the left side and 3 on the right side, and I draw in all possible allowed edges, how many edges does it have? What if there are m vertices on the left side and n on the right?

11. Find the maximum number of edges we can fit in a graph on n vertices without creating any triangles as subgraphs.

12. Let's say we are allowed to make triangles in our graph, but we cannot have any complete graphs on 4 vertices, K_4, as subgraphs. What is the maximum number of edges we can make in our graph without making any copies of K_4?

An open problem!

The following problem is really hard. In fact, it is so hard that nobody in the world has solved it yet. Of course, it would be very nice to solve this open problem and get an exact maximum number of edges, but it would also be interesting to find upper and lower bounds for the maximum, or find examples of nice graphs that have a lot of edges without having the following forbidden subgraph:

13. How many edges can we draw in a graph on n vertices without creating the following forbidden subgraph:

Notice that this graph is bipartite. The forbidden subgraph problem seems to be very hard for bipartite graphs in particular. Also notice that this graph is just a funny drawing of a 4 cycle. Which cycles are bipartite?

14. Make your own adventure! Choose any pattern you can come up with–maybe a path of length 2, a cycle of length 5, a larger complete graph, any graph you can come up with that looks interesting–and find the number of edges you can put in a graph on n vertices if your graph is forbidden to be a subgraph. If you can't find the exact number, then as before, good upper and lower bounds or cool constructions of graphs that have a lot of edges without your forbidden pattern are also interesting to find!

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Groups and Permutations

by Girls' Angle Staff

For this Summer Fun problem set, we assume that you know the definition of a finite group. If not, check out page 17 of Robert Donley's "Permutations and Basic Group Theory: Part 1" in the previous issue of this Bulletin. In his "Permutations and Basic Group Theory: Part 2" (page 8), Robert discusses group actions on sets, which are maps from groups into the group of permutations of a set. Let's further explore the relationship between groups and permutations.

Let G be a finite group. Let $X = G$. Let $f : G \to S_X$ be the map that sends g in G to the permutation of X that sends x to gx.

1. Verify that $f(g)$ is, indeed, a permutation of X.

2. Show that f is a group homomorphism, that is, for any g and h in G, we have $f(gh) = f(g)f(h)$, where the product on the right side is composition of permutations.

3. Show that f is one-to-one, that is, for any g and h in G, if $f(g) = f(h)$, then $g = h$.

Problems 1-3 establish that f is a faithful group action. Thus, *every finite group can be realized as a subgroup of the group of permutations on some finite set*.

4. The permutations on {1, 2, 3} constitute the permutation group S_3. In cycle notation, its elements are: the identity element, (12), (23), (31), (123), (132). Let's label these elements 1 through 6. Apply the construction of f to get a realization of the elements of S_3 as elements of S_6.

The group action f establishes an upper bound on the size of a set on which the group has a faithful action (namely, the size of the group itself). Since S_3 acts naturally on the set {1, 2, 3}, we know that there can sometimes be faithful actions of a group on a set smaller than the group.

5. However, suppose G has p elements, where p is a prime number. Prove that there is no set X of size less than p on which G acts faithfully.

6. Even more, let G be addition, modulo p^n, where p is a prime number and n is a positive integer. Prove that there is no set X of size less than p^n on which G acts faithfully. (Note that Problem 5 is the case $n = 1$.)

7. Let G be addition, modulo 6. What is the size of the smallest set X on which G acts faithfully?

8. Let G be addition, modulo n, where n is an integer greater than 1.
What is the size of the smallest set X on which G acts faithfully?

9. Show that S_n cannot act faithfully on a set with fewer than n elements.

Summer Fun!

Notes from the Club

These notes cover some of what happened at Girls' Angle meets. In these notes, we include some of the things that you can try or think about at home or with friends. We also include some highlights and some elaborations on meet material. Less than 5% of what happens at the club is revealed here.

Session 34 - Meet 11 May 2, 2024	Mentors:	Elisabeth Bullock, Jade Buckwalter, Shauna Kwag, Bridget Li, Gautami Mudaliar, AnaMaria Perez, Jing Wang, Dora Woodruff, Saba Zerefa

One of our members designed and wove a friendship bracelet that illustrates counting in binary. Bracelet patterns are a rich source of mathematics. Although different from friendship bracelets, check out Hanna Mularczyk's Summer Fun problem set about braiding on page 18. And here's another type of bracelet-related math problem: Suppose you make a bracelet by stringing together 29 beads into a loop. You use two types of beads. How many different bracelet patterns are possible?

Session 34 - Meet 12 May 9, 2024	Mentors:	Elisabeth Bullock, Alexandra Fehnel, Bridget Li, Gautami Mudaliar, Padmasini Venkat, Dora Woodruff, Saba Zerefa

We held our traditional end-of-session math collaboration. This session's math collaboration was designed by Gautami Mudaliar and AnaMaria Perez, who came up with a clever error-correcting coding scheme for cracking the lock to our brand-new treasure chest! Designed and built by math teacher Gus Means, it looks like something that came off the Jolly Roger. Thank you, Gus!

Try your hand at solving some of the event's math problems:

Two tadpoles are racing. Only one tadpole can move at a time, and the first can move 1 step at a time while the other moves 2 steps at a time. How many ways can the first win a race of length 4?

If WOMP = 5115 and W, O, M, and P are all integers greater than 1, what is the value of the largest of W, O, M, and P?

How many of the first 500 Fibonacci questions are divisible by 11?

Our new treasure chest, designed and built by Gus Means.

A poet wishes to compose a 6-line poem where there are 3 pairs of rhyming lines and different pairs do not rhyme with each other. How many rhyming schemes are possible?

Calendar

Session 34: (all dates in 2024)

February	1	Start of the thirty-fourth session!
	8	
	15	
	22	No meet
	29	
March	7	
	14	
	21	
	28	No meet
April	4	
	11	
	18	No meet
	25	Support Network Visitor: Cecilia Esterman, Avangrid
May	2	
	9	

Session 35: (all dates in 2024)

September	12	Start of the thirty-fifth session!
	19	
	26	
October	3	
	10	
	17	
	24	
	31	
November	7	
	14	
	21	
	28	Thanksgiving - No meet
December	5	

Girls' Angle has run over 150 Math Collaborations. Math Collaborations are fun, fully collaborative, math events that can be adapted to a variety of group sizes and skill levels. We now have versions where all can participate remotely. We have now run four such "all-virtual" Math Collaboration. If interested, contact us at girlsangle@gmail.com. For more information and testimonials, please visit www.girlsangle.org/page/math_collaborations.html.

Girls' Angle can offer custom math classes over the internet for small groups on a wide range of topics. Please inquire for pricing and possibilities. Email: girlsangle@gmail.com.

Girls' Angle: A Math Club for Girls

Membership Application

Note: If you plan to attend the club, you only need to fill out the Club Enrollment Form because all the information here is also on that form.

Applicant's Name: (last) ______________________________ (first) ______________________________

Parents/Guardians: ____

Address (the Bulletin will be sent to this address):

Email:

Home Phone: ____________________________ Cell Phone: ____________________________

Personal Statement (optional, but strongly encouraged!): Please tell us about your relationship to mathematics. If you don't like math, what don't you like? If you love math, what do you love? What would you like to get out of a Girls' Angle Membership?

The $50 rate is for US postal addresses only. **For international rates, contact us before applying.**

Please check all that apply:

- ☐ Enclosed is a check for $50 for a 1-year Girls' Angle Membership.
- ☐ I am making a tax-free donation.

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com.

Girls' Angle

A Math Club for Girls

Girls' Angle
Club Enrollment

Gain confidence in math! Discover how interesting and exciting math can be! Make new friends!

The club is where our in-person mentoring takes place. At the club, girls work directly with our mentors and members of our Support Network. To join, please fill out and return the Club Enrollment form. Girls' Angle Members receive a significant discount on club attendance fees.

Who are the Girls' Angle mentors? Our mentors possess a deep understanding of mathematics and enjoy explaining math to others. The mentors get to know each member as an individual and design custom tailored projects and activities designed to help the member improve at mathematics and develop her thinking abilities. Because we believe learning follows naturally when there is motivation, our mentors work hard to motivate. In order for members to see math as a living, creative subject, at least one mentor is present at every meet who has proven and published original theorems.

What is the Girls' Angle Support Network? The Support Network consists of professional women who use math in their work and are eager to show the members how and for what they use math. Each member of the Support Network serves as a role model for the members. Together, they demonstrate that many women today use math to make interesting and important contributions to society.

What is Community Outreach? Girls' Angle accepts commissions to solve math problems from members of the community. Our members solve them. We believe that when our members' efforts are actually used in real life, the motivation to learn math increases.

Who can join? Ultimately, we hope to open membership to all women. Currently, we are open primarily to girls in grades 5-12. We welcome *all girls* (in grades 5-12) regardless of perceived mathematical ability. There is no entrance test. Whether you love math or suffer from math anxiety, math is worth studying.

How do I enroll? You can enroll by filling out and returning the Club Enrollment form.

How do I pay? The cost is $20/meet for members and $30/meet for nonmembers. Members get an additional 10% discount if they pay in advance for all 12 meets in a session. Girls are welcome to join at any time. The program is individually focused, so the concept of "catching up with the group" doesn't apply.

Where is Girls' Angle located? Girls' Angle is based in Cambridge, Massachusetts. For security reasons, only members and their parents/guardian will be given the exact location of the club and its phone number.

When are the club hours? Girls' Angle meets Thursdays from 3:45 to 5:45. For calendar details, please visit our website at www.girlsangle.org/page/calendar.html or send us email.

Can you describe what the activities at the club will be like? Girls' Angle activities are tailored to each girl's specific needs. We assess where each girl is mathematically and then design and fashion strategies that will help her develop her mathematical abilities. Everybody learns math differently and what works best for one individual may not work for another. At Girls' Angle, we are very sensitive to individual differences. If you would like to understand this process in more detail, please email us!

Are donations to Girls' Angle tax deductible? Yes, Girls' Angle is a 501(c)(3). As a nonprofit, we rely on public support. Join us in the effort to improve math education! Please make your donation out to **Girls' Angle** and send to Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038.

Who is the Girls' Angle director? Ken Fan is the director and founder of Girls' Angle. He has a Ph.D. in mathematics from MIT and was a Benjamin Peirce assistant professor of mathematics at Harvard, a member at the Institute for Advanced Study, and a National Science Foundation postdoctoral fellow. In addition, he has designed and taught math enrichment classes at Boston's Museum of Science, worked in the mathematics educational publishing industry, and taught at HCSSiM. Ken has volunteered for Science Club for Girls and worked with girls to build large modular origami projects that were displayed at Boston Children's Museum.

Who advises the director to ensure that Girls' Angle realizes its goal of helping girls develop their mathematical interests and abilities? Girls' Angle has a stellar Board of Advisors. They are:

Connie Chow, founder and director of the Exploratory
Yaim Cooper, Institute for Advanced Study
Julia Elisenda Grigsby, professor of mathematics, Boston College
Kay Kirkpatrick, associate professor of mathematics, University of Illinois at Urbana-Champaign
Grace Lyo, assistant dean and director teaching & learning, Stanford University
Lauren McGough, postdoctoral fellow, University of Chicago
Mia Minnes, SEW assistant professor of mathematics, UC San Diego
Beth O'Sullivan, co-founder of Science Club for Girls.
Elissa Ozanne, associate professor, University of Utah School of Medicine
Kathy Paur, Kiva Systems
Bjorn Poonen, professor of mathematics, MIT
Liz Simon, graduate student, MIT
Gigliola Staffilani, professor of mathematics, MIT
Bianca Viray, associate professor, University of Washington
Karen Willcox, Director, Oden Institute for Computational Engineering and Sciences, UT Austin
Lauren Williams, professor of mathematics, Harvard University

At Girls' Angle, mentors will be selected for their depth of understanding of mathematics as well as their desire to help others learn math. But does it really matter that girls be instructed by people with such a high-level understanding of mathematics? We believe YES, absolutely! One goal of Girls' Angle is to empower girls to be able to tackle *any* field regardless of the level of mathematics required, including fields that involve original research. Over the centuries, the mathematical universe has grown enormously. Without guidance from people who understand a lot of math, the risk is that a student will acquire a very shallow and limited view of mathematics and the importance of various topics will be improperly appreciated. Also, people who have proven original theorems understand what it is like to work on questions for which there is no known answer and for which there might not even be an answer. Much of school mathematics (all the way through college) revolves around math questions with known answers, and most teachers have structured their teaching, whether consciously or not, with the knowledge of the answer in mind. At Girls' Angle, girls will learn strategies and techniques that apply even when no answer is known. In this way, we hope to help girls become solvers of the yet unsolved.

Also, math should not be perceived as the stuff that is done in math class. Instead, math lives and thrives today and can be found all around us. Girls' Angle mentors can show girls how math is relevant to their daily lives and how this math can lead to abstract structures of enormous interest and beauty.

Girls' Angle: Club Enrollment Form

Applicant's Name: (last) ______________________________ (first) ______________________________

Parents/Guardians: ____

Address: ____ Zip Code: _________

Home Phone: _________________ Cell Phone: _________________ Email: ______________________

Please fill out the information in this box.

Emergency contact name and number: ____

Pick Up Info: For safety reasons, only the following people will be allowed to pick up your daughter. Names:

____

Medical Information: Are there any medical issues or conditions, such as allergies, that you'd like us to know about?

____

Photography Release: Occasionally, photos and videos are taken to document and publicize our program in all media forms. We will not print or use your daughter's name in any way. Do we have permission to use your daughter's image for these purposes? **Yes** **No**

Eligibility: Girls roughly in grades 5-12 are welcome. Although we will work hard to include every girl and to communicate with you any issues that may arise, Girls' Angle reserves the discretion to dismiss any girl whose actions are disruptive to club activities.

Personal Statement (optional, but strongly encouraged!): We encourage the participant to fill out the optional personal statement on the next page.

Permission: I give my daughter permission to participate in Girls' Angle. I have read and understand everything on this registration form and the attached information sheets.

____ Date: _______________________

(Parent/Guardian Signature)

Participant Signature: ____

Members: Please choose one.	Nonmembers: Please choose one.
☐ Enclosed is \$216 for one session (12 meets) ☐ I will pay on a per meet basis at \$20/meet.	☐ I will pay on a per meet basis at \$30/meet. ☐ I'm including \$50 to become a member, and I have selected an item from the left.

☐ I am making a tax-free donation.

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com. Also, please sign and return the Liability Waiver or bring it with you to the first meet.

Personal Statement (optional, but strongly encouraged!): This is for the club participant only. How would you describe your relationship to mathematics? What would you like to get out of your Girls' Angle club experience? If you don't like math, please tell us why. If you love math, please tell us what you love about it. If you need more space, please attach another sheet.

Girls' Angle: A Math Club for Girls

Liability Waiver

I, the undersigned parent or guardian of the following minor(s)

____,

do hereby consent to my child(ren)'s participation in Girls' Angle and do forever and irrevocably release Girls' Angle and its directors, officers, employees, agents, and volunteers (collectively the "Releasees") from any and all liability, and waive any and all claims, for injury, loss or damage, including attorney's fees, in any way connected with or arising out of my child(ren)'s participation in Girls' Angle, whether or not caused by my child(ren)'s negligence or by any act or omission of Girls' Angle or any of the Releasees. I forever release, acquit, discharge and covenant to hold harmless the Releasees from any and all causes of action and claims on account of, or in any way growing out of, directly or indirectly, my minor child(ren)'s participation in Girls' Angle, including all foreseeable and unforeseeable personal injuries or property damage, further including all claims or rights of action for damages which my minor child(ren) may acquire, either before or after he or she has reached his or her majority, resulting from or connected with his or her participation in Girls' Angle. I agree to indemnify and to hold harmless the Releasees from all claims (in other words, to reimburse the Releasees and to be responsible) for liability, injury, loss, damage or expense, including attorneys' fees (including the cost of defending any claim my child might make, or that might be made on my child(ren)'s behalf, that is released or waived by this paragraph), in any way connected with or arising out of my child(ren)'s participation in the Program.

Signature of applicant/parent: ____Date: ___________________

Print name of applicant/parent: ____

Print name(s) of child(ren) in program: ____