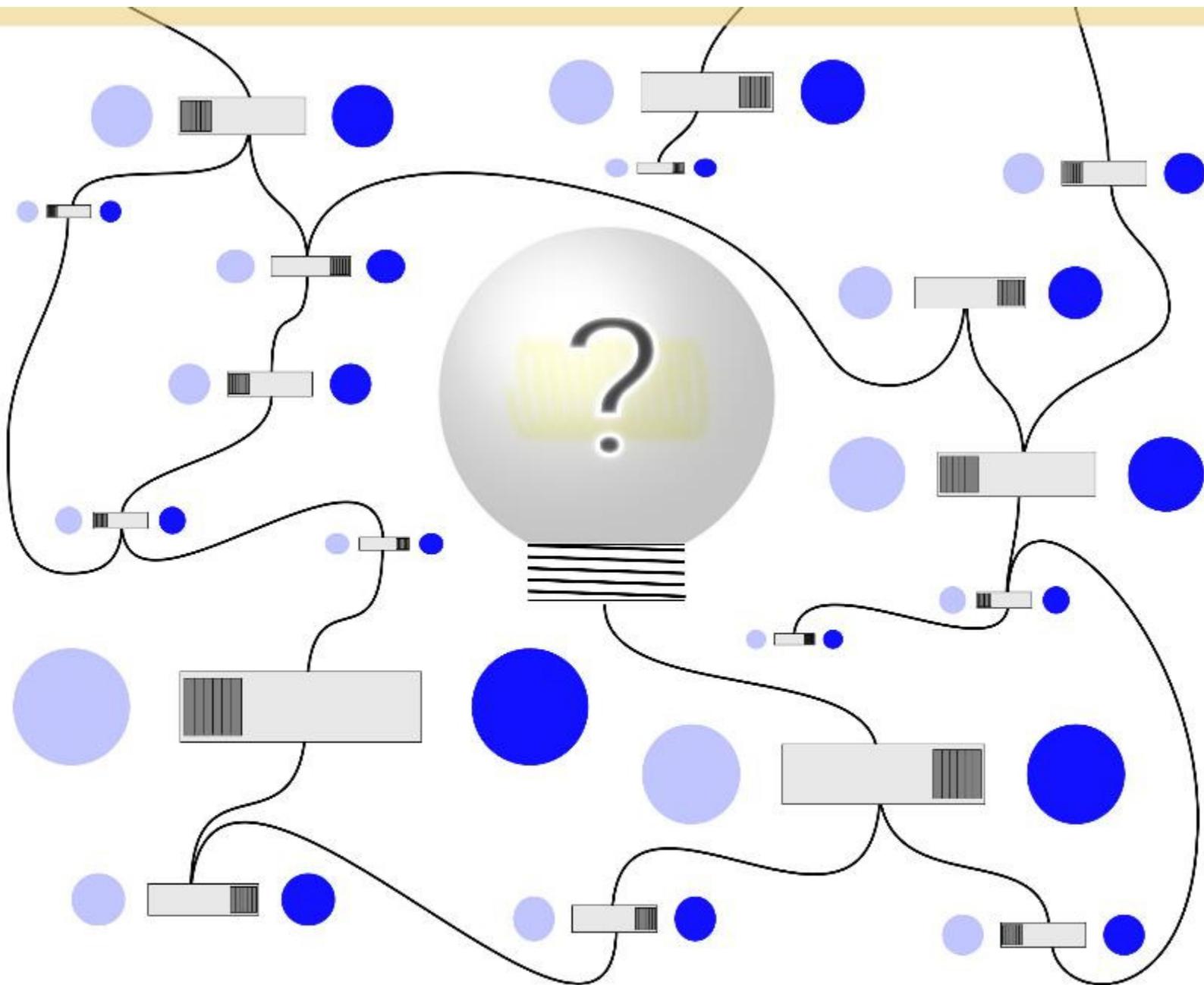


# Girls' *Angle* Bulletin

October/November 2020 • Volume 14 • Number 1

*To Foster and Nurture Girls' Interest in Mathematics*



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## From the Founder

A huge *Thank You* to our mentors, members, members' parents, tech crew, Kaylie Hausknecht and Phoebe Meyerson, and our Head Mentor, Grace Work, for their infinite patience and heroic efforts to keep the math alive through the challenges of the pandemic. - Ken Fan, President and Founder

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## Girls' Angle Bulletin

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On the cover: *Decisions, Decisions!* by C. Kenneth Fan. See page 13.

# An Interview with Pamela E. Harris, Part 1

Pamela E. Harris is an Associate Professor of Mathematics at Williams College. She obtained her PhD in Mathematics from the University of Wisconsin-Milwaukee under the supervision of Jeb Willenbring.

**Ken:** Thank you so much for doing this interview, and thank you for your wonderful two-part article contribution “Partitions from Mars,” which you co-authored with Alex Pankhurst, Cielo Perez, and Aesha Siddiqui.<sup>1</sup> Could you please tell us about how that paper came about and how your collaboration worked?

**Pamela:** That paper was definitely a lot of fun to work on! That work started in 2016 with a collaboration between Aesha and Cielo which began as part of a research course I taught at Williams College. It was a course on representation theory and we focused on combinatorial questions stemming from finding closed formulas for Kostant’s weight multiplicity formula. As we detail in the paper, that formula involves Kostant’s partition function. The following fall I taught a combinatorics class in which I introduced vector partition functions and that’s when Alex joined us on this project. The idea was, “How would you explain the concept of vector partition functions to (at the time) my 11-year-old daughter.” So the first plan was to make this more of a money exchange problem: Given a certain amount of money, in how many



Photo courtesy of Pamela Harris

ways could we have made that amount using coins with certain values. But that wasn’t quite right. Since money is exchangeable and we needed to think about having two objects and not being able to exchange them for a third. That’s where this clever idea we describe in the paper came from and which takes us on a visit to a Martian arcade and trading certain tickets where the rules don’t allow us to exchange any number of tickets for a different type of ticket. Once we had that setting, we spent a few weeks drafting the article and editing it. We also thought about a question that Jesus De Loera posed to me in December of 2017, which led us to understand more details of his work (in collaboration with Bernd Sturmfels) on Kostant’s partition function. That is where the second part of Partitions from Mars began.

<sup>1</sup> See pages 7-11 of Volume 11, Number 2 and pages 7-10 of Volume 11, Number 3 of the Girls’ Angle Bulletin.



Anthony Simpson, Cielo Perez, and Aesha Siddiqui at SACNAS

Photo courtesy of Pamela Harris

**Ken:** You do a lot of work to promote minorities in mathematics. Could you please tell us about your efforts? What motivates you?

**Pamela:** The first time I met a Latina mathematician was the penultimate year of my PhD. This really was quite sad because I went my entire undergrad career and most of my graduate school education without ever having actually met another Latina mathematician. During those years I knew that like I lacked a community of other Latinxs and Hispanics in mathematics. But I didn't realize how much having people to look up to and see myself reflected in would be life changing until I met Dr. Alejandra Alvarado.<sup>2</sup> She was someone I could speak in Spanish with, who shares the same heritage, who also loves math, and who had completed her PhD.

Having lived this first hand means that as an educator I am constantly aware of who appears on the covers of math

magazines/publications, who wins awards, who is featured as leading scientists and mathematicians of our time. As one may expect they tend to not be someone that looks like me. Yet there are plenty of minority mathematicians doing phenomenal work! So the problem stems from the lower visibility of their work. This realization has been the driving force of the work I undertake and which brings to light the contributions of Latinxs and Hispanics in the mathematical sciences.

**Ken:** You cofounded Lathisms.org. Please tell us about that.

**Pamela:** I mentioned before that I want to help bring to light the contributions of minority mathematicians. This is the goal of Lathisms: Latinxs and Hispanics in the Mathematical Sciences. We began by having a website celebrating Hispanic Heritage Month (which is celebrated yearly in the US from September 15-October 15). Every day we revealed a mathematician and provided a biography of their contributions. We later added podcast interviews with some of the Lathisms featured mathematicians. Now we have free-to-download posters as well. Our new goal is to highlight their *testimonios*, their personal stories and how they have succeeded. So we are currently finishing a book entitled *Testimonios: Stories of Latinxs and Hispanics in the mathematical sciences*. This book presents a collection of stories that intertwine how being Latinx/Hispanic has contributed to the success of those in the Lathisms community. The book is timely and much needed in the mathematical sciences. Now more than ever, the goal of Lathisms is to aim to bring to light those who have shaped, are shaping, and will

<sup>2</sup> Dr. Alejandra Alvarado is an Associate Professor of Mathematics at Eastern Illinois University and

received her doctoral degree in mathematics at the University of Arizona.

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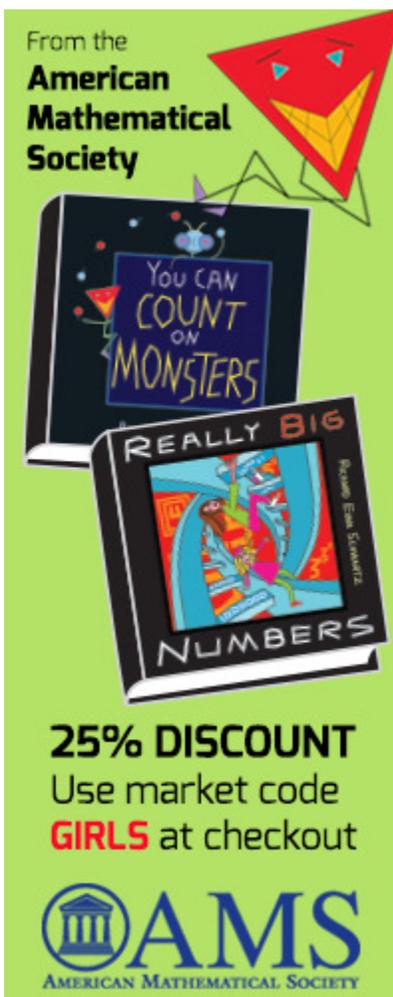
For this issue, those who do not subscribe to the print version will be missing out on a portion of this interview with Prof. Pamela E. Harris and other content. We hope that you consider the value of such content and decide that the efforts required to produce such content are worthy of your financial support.

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Thank you and best wishes,  
Ken Fan  
President and Founder  
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The American Mathematical Society is generously offering a 25% discount on the two book set *Really Big Numbers* and *You Can Count On Monsters* to readers of this Bulletin. To redeem, go to <http://www.ams.org/bookstore-getitem/item=MBK-84-90> and use the code “GIRLS” at checkout.



# The Needell in the Haystack<sup>1</sup>

Questions, Lyme Disease Data, and More Questions, Part 1  
by Deanna Needell<sup>2</sup> | edited by Jennifer Sidney Silva

Lyme disease affects me personally. In my January 2018 installment, I wrote about some machine learning work that my team had been doing with Lyme disease. Now, I bring some updates on this ongoing project while providing a tour of various basic machine learning methods that are useful in analyzing a certain type of data. I emphasize that here and always, data – and even the mathematical methods themselves – are full of bias and other practical challenges. I view this kind of work as exploratory. Rather than looking for answers, I seek interesting questions to pose to the scientific community.

Lyme disease is the most common vector-borne disease in the United States. The CDC estimates that 300,000 people in the U.S. are diagnosed with Lyme disease each year. A significant proportion of patients with Lyme disease develop chronic debilitating symptoms that persist in the absence of initial treatment or following short-course antibiotic therapy. It is estimated that as many as 36% of those diagnosed and treated early remain ill after treatment. However, despite the high incidence and severity of Lyme disease, little research has been done, both clinically and analytically. The result has been a stagnant and controversial research environment with minimal innovation and a costly lack of understanding or consensus. Physicians still do not know the best way to diagnose or treat Lyme, how it progresses, or why some patients respond to treatment and others do not.

Can machine learning help both physicians and patients to find better treatments and identify factors that might predict treatment response?

## The Data

Founded over 30 years ago, LymeDisease.org (LDo) is a national non-profit dedicated to advocacy, research, and education. In November 2015, LDo launched MyLymeData, a patient registry, which has since enrolled over 13,000 patients and continues to grow. Participants are asked hundreds of questions regarding their health history, diagnosis, symptoms, and treatment.

The first study using data from the registry was published in 2018. That study focused on treatment response variation among patients and identified a subgroup of high treatment responders using the **Global Rating of Change Scale (GROC)**, a widely used and highly regarded treatment response measurement. The GROC survey questions assess the degree to which participants report that their condition improved, worsened, or remained unchanged following antibiotic treatment.

For our work, we restricted our dataset to participants who met the following criteria:

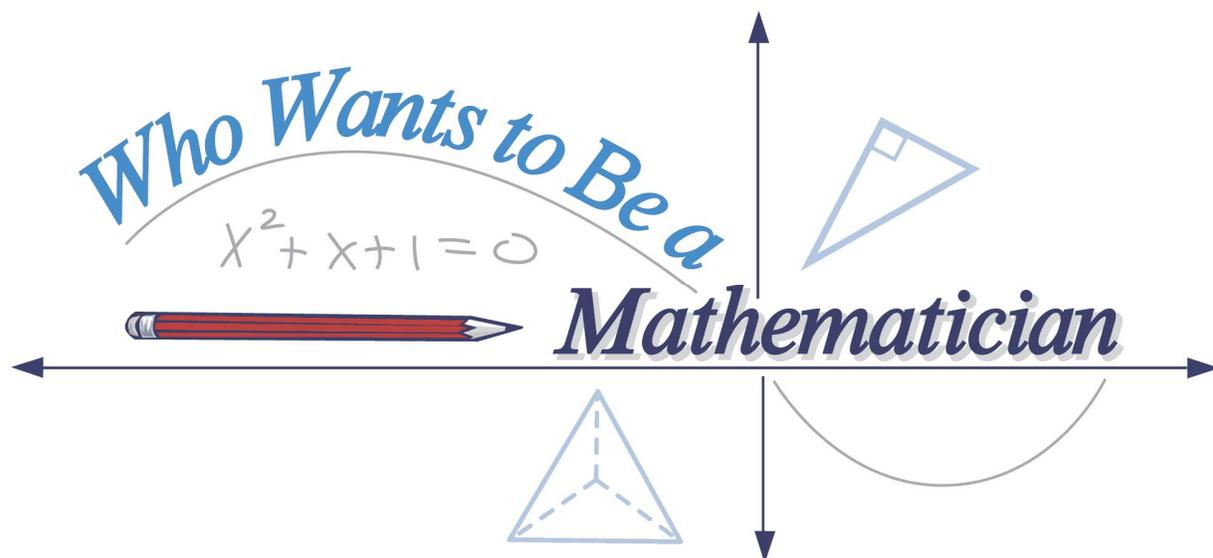
- (1) They were US residents with chronic Lyme disease (i.e., patients who have experienced persistent symptoms for at least six months after antibiotic treatment).
- (2) They responded that they were unwell.
- (3) They answered the GROC survey questions.

---

<sup>1</sup> This content supported in part by a grant from MathWorks.

<sup>2</sup> The author thanks her student, Josh Vendrow, her postdoctoral scholar, Dr. Jamie Haddock, and the CEO of LymeDisease.org, Lorraine Johnson, for their help with this project.

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**SVM** In Figure 2, we record the change in training accuracy of the SVM on SMLD after removing each feature from the data set individually. In order to accurately assess the effect of dropping each feature, we take random subsets of SMLD and run the model on each subset of the data. We suggest that a larger decrease in accuracy when dropping a feature demonstrates that feature’s ability to predict GROC value.

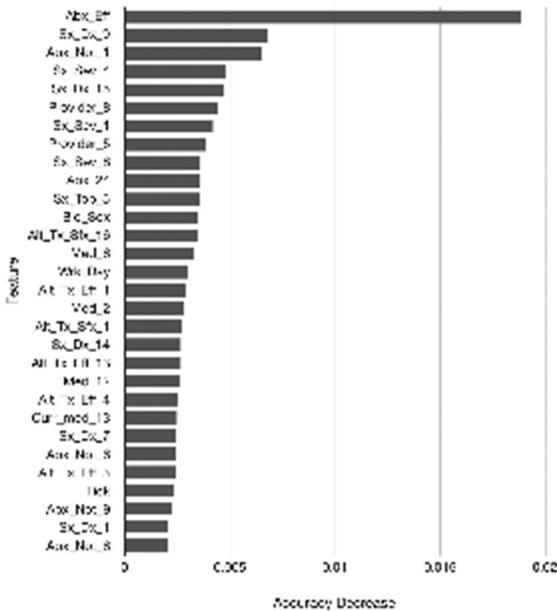


Figure 2. SVM training accuracy dropping individual features on Subsampled MLD (SMLD). For each feature, we measure the decrease in accuracy caused by removing a single feature from the dataset. Here we display the top 30 features by accuracy in descending order.

In Figure 3, we show the results of running the SVM model on only individual features, rather than dropping the feature from the entire dataset. In both figures we again see that antibiotic-related questions are playing an important role, as are symptom-related questions. Interestingly, in Figure 2, we even see some questions about the type of physician the patient sees playing a role.

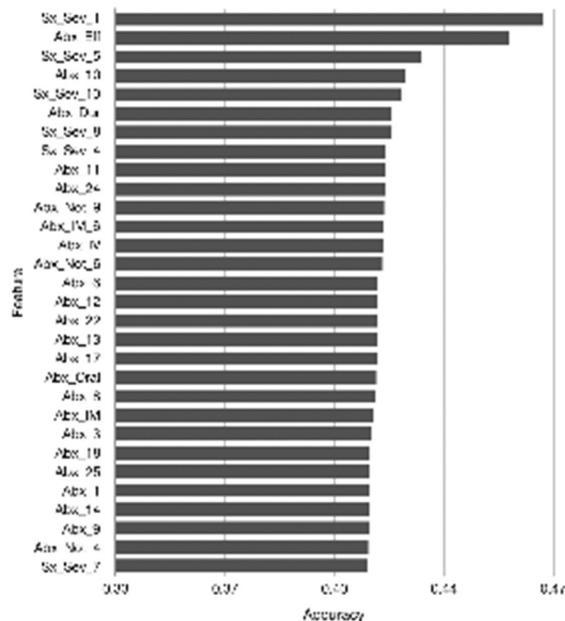


Figure 3. SVM accuracy of individual features on Subsampled MLD (SMLD). We attain this accuracy by running our SVM model on only the single feature. Here we display the top 30 features by accuracy in descending order.

We’ll look at the results of the other methods in Part 2.

# Switcheroo!

by Lightning Factorial | edited by Amanda Galtman



Your task: Determine if the light bulb is on or off.

## The Setup

There's a light bulb and a bank of a thousand switches, numbered 1 through 1000.

Each switch has two settings: Lavender and Blue.

The light bulb is controlled by Switch 1. If Switch 1 is on, then the light bulb is on, but if Switch 1 is off, the light bulb is off.

But when is a switch considered to be on and when is it off? Here's where it gets tricky.

Normally, Lavender corresponds to on and Blue corresponds to off.

However, sometimes a switch is influenced by another switch. If Switch  $B$  is influenced by Switch  $A$ , then when  $A$  is on, the meaning of Lavender and Blue are swapped for Switch  $B$ . It's possible for one switch to influence several switches, and it is also possible for a switch to be influenced by several switches. If a switch is influenced by several switches, the effect is cumulative. That is, if two switches are on and both swap the meaning of Lavender and Blue for a certain Switch  $X$ , then the meaning of Lavender and Blue for Switch  $X$  are swapped twice. In effect, that means that for Switch  $X$ , Lavender means on and Blue means off, as they normally do.

All that remains to explain is how the switches influence each other.

## Scenario 1

In our first scenario, for any  $N > 1$ , if Switch  $N$  is on, then it swaps the meaning of Lavender and Blue for Switch  $N - 1$ , that is, Switch  $N$  influences Switch  $N - 1$ . Switch 1 does not influence any switch. It only affects the light bulb.

For example, suppose that Switches 1, 2, and 5 are set to Lavender, while all the other switches are set to Blue. Is the light bulb on or off?

Since Switches 6-1000 are set to Blue, and no switch influences a switch with a higher label, those switches are all off. Hence, Switch 5 behaves normally, and since it is set to Lavender, it is on. Switch 5 influences Switch 4, which means that the meanings of Lavender and Blue for Switch 4 are swapped so that Lavender means off and Blue means on. Since Switch 4 is set to Blue, it is therefore on. Switch 4 influences Switch 3, and so the meanings of Lavender and Blue for Switch 3 are swapped, too. Since Switch 3 is set to Blue, it is therefore on. Switch 3 influences Switch 2, so the meanings of Lavender and Blue for Switch 2 are swapped, and since Switch 2 is set to Lavender, it is therefore off. That means that the meanings of Lavender and Blue for Switch 1 are unaffected, and since Switch 1 is set to Lavender, it is therefore on. Hence, the light bulb is shining brightly!

Got it?

1. For which numbers  $N$  will the light bulb be on if switch  $N$  is set to Lavender while all the other switches are set to Blue?
2. If all the even-numbered switches are set to Lavender while all the odd-numbered switches are set to Blue, is the light bulb on or off?
3. If all the odd-numbered switches are set to Lavender while all the even-numbered switches are set to Blue, is the light bulb on or off?
4. If Switch  $N$  is set to Lavender when  $N$  is a perfect square and Blue otherwise, is the light bulb on or off?
5. Can you develop a criterion that enables you to quickly determine whether the light bulb is on or off, given any arrangement of the switches being set to Lavender and Blue?
6. Can you solve Problem 5 if there are  $N$  switches instead of 1000, for any positive integer  $N$ ?

## Scenario 2

In our second scenario, Switch  $N$  influences Switch  $K$  if and only if  $K < N$ . So, unlike Scenario 1, Switch  $N$  influences  $N - 1$  switches and is influenced by  $1000 - N$  switches.

For example, suppose that Switches 1, 2, and 5 are set to Lavender, while all the other switches are set to Blue. Is the light bulb on or off?

As in Scenario 1, no switch influences a switch with a higher label, and since Switches 6-1000 are set to Blue, they are therefore all off. Hence, Switch 5 behaves normally, and since it is set to Lavender, it is on. Switch 5 influences Switch 1, Switch 2, Switch 3, and Switch 4. That means that the meanings of Lavender and Blue for Switches 1-4 are swapped at least once. Since Switch 4 is influenced by Switches 5-1000 and we know that Switch 5 is on while Switches 6-1000 are all off, we know that for Switch 4, the cumulative effect is that Lavender now means off and Blue now means on. Since Switch 4 is set to Blue, it is therefore on. Switch 4 influences Switch 1, Switch 2, and Switch 3. We now know the on/off status of all switches that influence

Switch 3. There is a cumulative effect of two swaps because Switch 4 and Switch 5 are on, but Switches 6-1000 are off. Therefore, Switch 3 behaves normally. Since Switch 3 is set to Blue, it is therefore off, which means it doesn't affect other switches. Hence, Switch 2 also has had the meaning of Lavender and Blue swapped twice (by Switches 4 and 5, which are the only switches with a label bigger than 2 that are on), and, hence, operates normally. Since Switch 2 is set to Lavender, it is on. Switch 2 influences only one switch, Switch 1. Reviewing the status of Switches 2-1000, we find that the meanings of Lavender and Blue for Switch 1 have been swapped a total of three times (by Switches 2, 4, and 5). The net effect is that for Switch 1, Lavender means off and Blue means on. Since Switch 1 is set to Lavender, it is off. Therefore, the light bulb is dark.

7. Redo Problems 1-6 for Scenario 2 in place of Scenario 1.

We'll now describe both Scenarios 3 and 4.

### Scenario 3

In our third scenario, let  $N$  and  $K$  be positive integers. Switch  $N$  influences Switch  $K$  if and only if  $K < N$  and when you subtract  $K$  from  $N$  in binary, there are no carries. Another way of putting it is that Switch  $N$  influences Switch  $K$  if and only if  $K < N$  and if you write both  $N$  and  $K$  in binary, whenever there is a 1 in the  $2^d$  place of the binary representation of  $K$ , there is also a 1 in the  $2^d$  place of the binary representation of  $N$ .

For example, suppose that Switches 1, 2, and 5 are set to Lavender, while all the other switches are set to Blue. Is the light bulb on or off?

This time, rather than working through this example in detail, we will only say that the answer is that the light bulb is off. If you would like one more example to check that you have the rules for this scenario down, we offer that the light bulb is shining brightly if Switches 1, 2, 3, and 5 are set to Lavender, while all the other switches are set to Blue.

### Scenario 4

In our fourth and last scenario, let  $N$  be a positive integer. If Switch  $N$  is on, then it swaps the meaning of Lavender and Blue for proper divisors  $K$  of  $N$ . In other words, if Switch  $N$  is on, it swaps the meaning of Lavender and Blue for Switch  $K$  if and only if  $K < N$  and  $K$  divides evenly into  $N$ .

For example, suppose that Switches 1, 2, and 5 are set to Lavender, while all the other switches are set to Blue. Is the light bulb on or off?

Again, rather than working through this example in detail, we will only say that the answer is that the light bulb is on.

8. Redo Problems 1-6 for Scenarios 3 and 4.

9. Invent your own Scenario by describing which switches influences which switches.

10. Does it make sense to allow a switch to influence itself?

# Number Search

by Addie Summer

Here are a bunch of grids consisting of decimal digits. Your task is to find numbers in these grids that fit into the requested category. Just as in word search puzzles, numbers consist of consecutive digits found in the grid and may go forward, backward, up, and down, as well as in all four diagonal directions. (Don't count numbers with leading zeroes.)

Find all 35 positive multiples of 18.

---

8	9	4	3	2	1
6	3	6	2	7	9
4	4	7	8	0	4
5	8	3	6	0	6
2	0	0	1	0	1
0	4	5	2	7	4

---

1	4	4	4
8	6	4	9
2	1	1	6
1	6	8	1

Find all 32 perfect squares greater than 10.

Find all 25 powers of 2 greater than 10.

---

1	2	5	6	4	5	1	6
5	3	1	4	2	3	8	1
3	2	0	6	3	6	0	3
1	9	7	1	0	1	4	4
9	9	5	1	7	7	7	1
1	2	1	6	9	9	3	1
8	0	2	0	6	6	2	8
2	3	0	6	9	1	6	8

---

5	0	3	8	4	8
3	0	6	9	2	7
5	0	4	7	2	7
0	6	9	8	2	2
2	4	2	0	2	0
2	1	2	8	0	2

---

Find all 39 prime numbers greater than 10.

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# Make a Toothpick Icosahedron

by Emily and Jasmine | edited by Amanda Galtman

At the beginning of *The Water Column* (see page 22), we were in the midst of building a regular icosahedron with toothpicks. We finished it, and that's a photo of the finished model to the right.



We thought we'd explain how we made it in case you want to make one for yourself. All you'll need are 30 toothpicks, some glue, and a good measure of patience.

## Materials

We picked out the straightest 30 toothpicks we could find and made sure that they all had the same length. We used round toothpicks instead of flat ones because we wanted the ends of the toothpicks to be identical, but this construction works with flat toothpicks, too.

We used Elmer's glue, but any wood glue should work. If it's easy to apply and doesn't take too long to set, that's even better.

## The Regular Icosahedron

An icosahedron is a solid with 20 triangular faces, 30 edges, and 12 vertices. Every vertex is the endpoint of 5 edges and the common vertex of 5 faces. An icosahedron is **regular** if all of its faces are equilateral triangles. The regular icosahedron is the Platonic solid with the greatest number of faces, and it enjoys many beautiful properties. For example, opposite edges form the short sides of a golden rectangle, and the solid's 12 vertices can be grouped into 3 sets of 4, where each set of 4 forms the vertices of such a golden rectangle. Even more, the 3 resulting golden rectangles are mutually perpendicular. If you build this model, you'll see these facts.

At this point, why don't you try to figure out your own way to build a regular icosahedron? Ours is not the only way, and you might come up with something better. (Send us a pic!)

## The Basic Strategy

There's essentially only one way to glue the ends of three toothpicks together to form a triangle, and the resulting triangle will be rigid. Also, certain groups of 5 toothpicks in a regular icosahedron form flat regular pentagons, such as the ones colored blue in Figure 1. Flat objects have the advantage that we can build them on a table without using external supports.

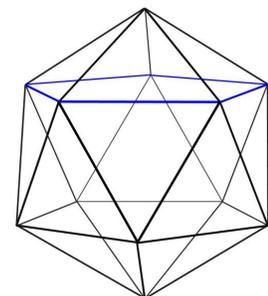


Figure 1

Our strategy was to exploit flatness and the many triangles, while also taking advantage of the stability of tripods.

## Step 1

We began by building the blue regular pentagon in Figure 1. To do that, we used the fact that the interior angles of a regular pentagon all measure  $108^\circ$ . We used a computer to make an image of a  $108^\circ$  angle and printed that to make a template. We created that angle with toothpicks by placing two toothpicks directly on the template and joining them with a dab of glue. When that dried, we repeated the process until we had a toothpick regular pentagon.

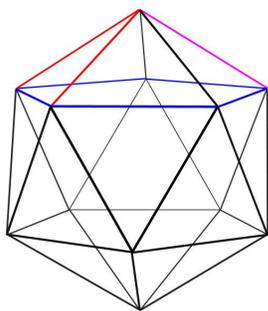


Figure 2

## Step 2

Next, we built a pentagonal pyramid using our toothpick regular pentagon as the base.

To do this, we first made a  $60^\circ$  angle with two toothpicks by forming an equilateral triangle with three toothpicks, but only adding glue at one of the vertices. We then took this  $60^\circ$  angle and another toothpick and added them to the pentagonal base, leaning them against each other. (See the red  $60^\circ$  angle and the purple toothpick in Figure 2.) We then completed the pentagonal pyramid by gluing in place the remaining two edges.

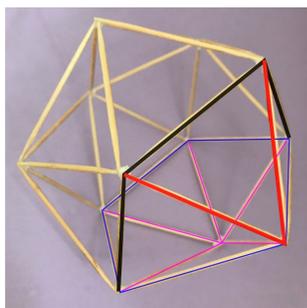


Figure 3

## Step 3

This was probably the trickiest step. If you can get past this step, you've made it over the hill. (This is where Emily was at the beginning of *The Water Column*.)

First, we connected 3 sides of a regular pentagon, by building a regular pentagon as in Step 1 but stopping after we'd connected just 3 of the edges. Also, we formed another  $60^\circ$  angle with two toothpicks.

We placed the pentagonal pyramid onto one of its lateral faces. In Figure 3, which shows the icosahedron from above, the blue and purple edges correspond to this pentagonal pyramid. We added the 3 connected sides of the regular pentagon (shown in black) and the  $60^\circ$  angle (shown in red) to the model. We had to hold these pieces in place until the glue dried enough to hold them in place for us.

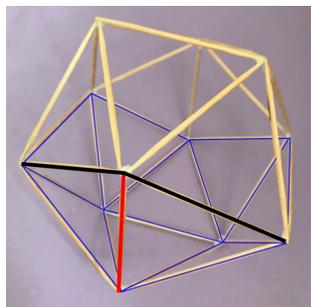


Figure 4

## Step 4

From here on, we simply added edges in whatever manner seemed stable. For example, in Figure 4, the blue edges represent the part of the model made in Steps 1-3. We then made a  $108^\circ$  angle (in black in Figure 4) using our pentagon angle template and added that to our model together with the edge colored red.

After the last piece is added and the glue dries, you might be surprised by the model's strength. We can rest a textbook upon ours.

## Tips

As you build your model, do your best to make sure that toothpick tips meet toothpick tips.

To make it possible to bring toothpick tips together, as you build the model, place the dabs of glue inside the angle formed

by two toothpicks. If you place the tips so that they meet somewhere inside a blob of glue, when the glue dries, you won't be able to bring another toothpick tip to meet the other tips.

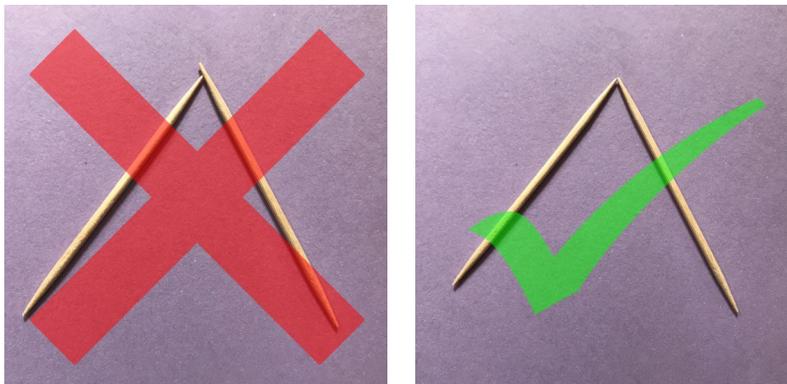


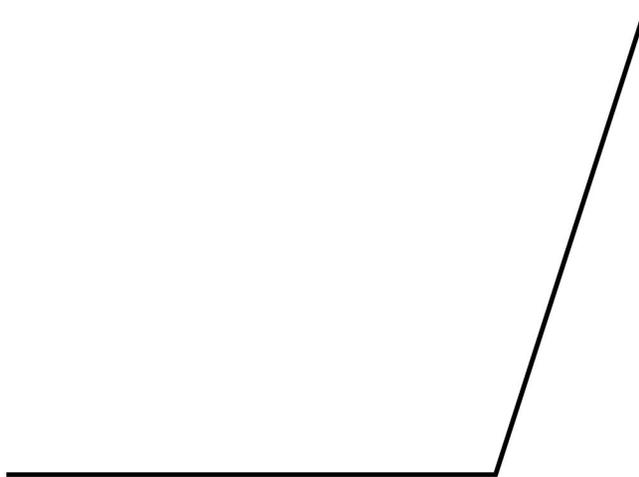
Figure 5. Make sure the toothpicks meet tip to tip.

Finally, be patient! Allow glue to dry thoroughly after each step, and plan to build this model over the course of several short sessions.

## Questions

1. Opposite vertices are the tips of pentagonal pyramids with disjoint pentagonal bases. Are these bases parallel to each other?

2. What is the shortest path connecting opposite vertices if you confine your travels to the surface of the icosahedron?



A 108° angle template.

3. Can you prove that opposite edges form the short sides of a golden rectangle? (A golden rectangle is a rectangle whose aspect ratio is the golden mean, which is  $\frac{1+\sqrt{5}}{2}$ .)

4. Can you find a formula for the surface area and volume of a regular icosahedron of side length  $s$ ? What is the measure of the dihedral angle between two adjacent faces of a regular icosahedron?

5. What are the volume, surface area, and height (when resting on a triangular face) of your toothpick icosahedron?

6. Can you make toothpick models of all the Platonic solids?

# The Water Column

by Ken Fan | edited by Jennifer Sidney Silva

Emily: Jasmine, can you get me a glass of water?

Jasmine looks over and sees Emily's hands stuck in an awkward position, deftly holding some toothpicks in place while dabs of glue firm up. She is making a model of an icosahedron.

Jasmine: Sure!

Jasmine gets up and heads to the kitchen. Emily is blowing on the glue, trying to get it to set faster. *Does blowing on glue make it set faster?* Jasmine wonders. All of a sudden, she hears water gushing from the kitchen faucet. Then, just as suddenly, she hears an adjustment in the water pressure. The water sounds like it is trickling out, then it increases again. These faucet adjustments continue for a bit, then settle to the sound of a steady, gentle stream.

*My lips feel so dry!* Emily thinks as she tries to let go of the toothpicks. They shift, so Emily quickly restores their position and holds them in place. *How long does this glue take to dry?*

A few minutes pass, and the tap water is still flowing, with no sign of Jasmine. *What's taking so long?* Emily wonders. *By now, she could have filled a bathtub!*

Emily tries to let go again.

Emily: Great! It's holding!

Emily gets up and heads to the kitchen. She sees Jasmine stooped over the sink, staring intently at the gentle flow of water from the tap.

Emily: Is something wrong with the water?

Jasmine: Not at all! It's beautiful!

Emily thinks wistfully about the last trip she took with her family before the pandemic. They visited the islands of Hawaii and marveled at the variety of forms water could take, from ocean waves at Waikiki, to the cascade at Waimea Falls, to the perpetual mist at Mt. Wai'ale'ale. She joins Jasmine, stoops over the kitchen sink, and stares at the gentle stream of water forming a translucent column against the metallic fixtures.

Emily: Not exactly a waterfall, but still strangely meditative.

Jasmine: Yes. And I was wondering exactly how this column narrows as the water falls. Oh, here's your glass of water!

Emily: Thanks! Hmm. The water doesn't form a geometric cylinder. It really does narrow. How curious. Let's try to figure it out!

Jasmine: Okay!

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Jasmine: Let's analyze the dimensions as a check to see if this formula is reasonable.

Emily: Well,  $b$  is the speed of the water just when it leaves the faucet, so that has dimensions length over time. And  $2a$  is the gravitational acceleration, so  $a$  has units length over time squared. Since  $x$  is a distance, the quantity  $b^2 + 4ax$  has units length squared over time squared.

Jasmine: And  $k$  has units of volume over time. So the whole expression under the first radical has units length cubed over time, divided by length over time, which is length squared. When we take the final square root, we end up with just length, so that checks out!

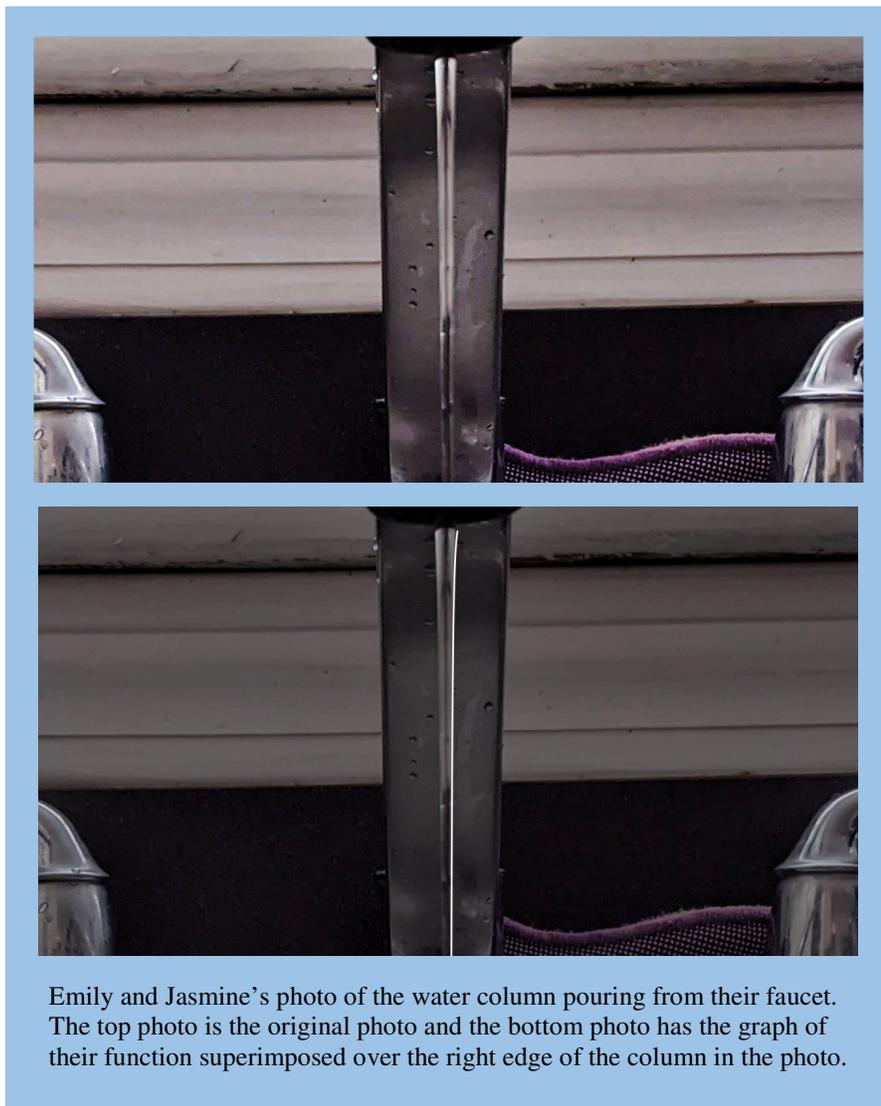
Emily: And as  $x$  grows,  $r(x)$  tends to 0, as it should. I think we found the formula!

Jasmine: Let's see if there are values of  $a$ ,  $b$ , and  $k$  that fit the column from the faucet.

Emily: Okay. In terms of curve fitting, we can always arrange it so that our units of measure make any one of the variables  $a$ ,  $b$ , or  $k$  equal to 1. That'll simplify things a little bit, and it means that we only have to sample two cross sections along the column.

Jasmine: Let's take a photo and use the width of a pixel in the photo as our unit of distance, then use whatever unit of time allows us to set  $b = 1$ .

Emily and Jasmine carry out their plan, using some graphic design software to determine the radii of two cross sections. They compute the values of  $a$  and  $k$  and plot the resulting function directly over their photo.



Emily and Jasmine's photo of the water column pouring from their faucet. The top photo is the original photo and the bottom photo has the graph of their function superimposed over the right edge of the column in the photo.

Emily: Wow, it fits to the accuracy of the pixels in this photo!

Jasmine: That was fun!

# Notes from the Club

These notes cover some of what happened at Girls' Angle meets. In these notes, we include some of the things that you can try or think about at home or with friends. We also include some highlights and some elaborations on meet material. Less than 5% of what happens at the club is revealed here.

Special thanks to our amazing Tech Crew, Kaylie Hausknecht and Phoebe Meyerson, for enabling our virtual meets to run so much more smoothly!

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Session 27 - Meet 1  
September 10, 2020

Mentors: Cecilia Esterman, Tina Lu, Laura Pierson, Kate Pearce, Nehar Poddar, Gisela Redondo, Melissa Sherman-Bennett, Christine Soh, Emma Wang, Jane Wang, Rebecca Whitman, Kailey Yang, Annie Yun, Angelina Zhang, Jasmine Zou

The Girls' Angle Club has been forced to go virtual for all of 2020-21. The virtual environment poses many new challenges. We are so thankful to our mentors, members, and all the parents who were so understanding and patient as we navigate this new, virtual environment. One benefit to moving the Girls' Angle meets online was that we could have members and mentors join who may not have been able to come to the in person meets. We also enjoyed getting to know eleven new members.

On occasion, to break the ice, we play mathematical games at the club. Head Mentor Grace Work invented a game that requires good communication from all participants. It is a collaborative variant on the "Describe This Drawing" game, which we've been playing for years as a way to develop pattern recognition, articulation, and attention to detail.<sup>3</sup> For the collaborative variant, a drawing of something readily recognizable is divided into pieces, none of which provide sufficient information to guess what the original drawing is. Each player is given one of the pieces. Players may look at their own piece, but are not allowed to see the other pieces or the final drawing. The players' goal is to reconstruct the original drawing. They may talk to each other as much as they wish, but they may not use gestures. The game is ideal for the Zoom app format. On Zoom, participants can turn off their video feeds for this game, but leave their audio on. From the mentor's perspective, we can get a good sense of how students think by the ways they choose to describe their piece of the drawing and by the questions they ask each other. We can also gain a sense of how they work together.

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Session 27 - Meet 2  
September 17, 2020

Mentors: Cecilia Esterman, Laura Pierson, Nehar Poddar, Gisela Redondo, Melissa Sherman-Bennett, Christine Soh, Emma Wang, Rebecca Whitman, Hanna Yang, Annie Yun, Angelina Zhang, Jasmine Zou

At Girls' Angle, we try hard to engage members in math that pertains to their preexisting interests, either by inducing them to wonder about something mathematical or indicating math relevant to the things they like to discuss. Recently, one of our members expressed interest in fictional stories with talking animals. By embracing this interest and making note of anything she said that had mathematical content, she ended up creating an interesting probability problem which has taken hold at the club, spawning several variants. Details will have to wait for a future Bulletin, but suffice it to say that it involves a voracious gorilla and an infinite supply of bananas!

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<sup>3</sup> See page 17 of the *Notes from the Club* for Session 20, Meet 1 in Volume 10, Number 3 of this Bulletin.

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Session 27 - Meet 3  
September 24, 2020

Mentors: Cecilia Esterman, Tina Lu, Laura Pierson, Kate Pearce,  
Nehar Poddar, Gisela Redondo, Emma Wang, Jane Wang,  
Rebecca Whitman, Annie Yun, Angelina Zhang

Have you heard of hand pulled noodles? It's a Chinese technique for making lots of noodles quickly.<sup>4</sup> One long noodle is made by repeatedly stretching and doubling over a specially prepared blob of dough. When the chef is satisfied, the folded ends are removed to produce individual strands. (You can watch chefs making hand pulled noodles on YouTube. Search for "hand pulled noodles" or for the Chinese term "lamian".) Suppose a chef can hand pull just over 1,000 noodles in a minute. How long would it take the chef to make over 1,000,000 noodles? Suppose the chef doubles the dough over 22 times and the length of the noodle between folds is 1 meter, how far would the noodle reach from your home town if completely unfolded? With each doubling, by what factor does the diameter of the noodle change? How many times would you have to double in order to make a noodle that could stretch from here to the moon?

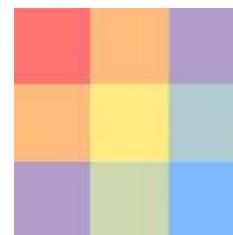
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Session 27 - Meet 4  
October 1, 2020

Mentors: Cecilia Esterman, Megan Joshi, Kate Pearce,  
AnaMaria Perez, Laura Pierson, Nehar Poddar,  
Gisela Redondo, Melissa Sherman-Bennett, Jane Wang,  
Rebecca Whitman, Hanna Yang

For our virtual meets, we frequently use the whiteboard provided by the Zoom app. One of our members noted that when you overlap two highlighter colors you get different colors in the overlapping region depending on the order in which the colors were placed over each other. This led to the question: How many different colors can be obtained using a Zoom whiteboard? At the time of this writing, Zoom whiteboards provide 15 predefined colors and do not allow the user to select arbitrary red, green, and blue values. If we only allow two colors to overlap, how many different colors are possible? What about when you allow 3 overlapping colors?

Exploring this question led to some that were more specific to the Zoom app, such as, how many times would you have to overlap a highlighter color on top of itself before it became identical to the solid color? What is the formula Zoom uses to produce the RGB values of the overlap, given the RGB values of the overlapping colors and the order that they are placed? If you created graphic processing software, what formula would you devise for combining the RGB values of overlapping, transparent colors?



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Session 27 - Meet 5  
October 8, 2020

Mentors: Cecilia Esterman, Kate Pearce, AnaMaria Perez,  
Laura Pierson, Nehar Poddar, Gisela Redondo,  
Emma Wang, Jane Wang, Rebecca Whitman, Annie Yun

Have you ever thought about what you could make with more than one set of tangrams? Each tangram set consists of 7 polygons that fit together to make a square. Given two tangram sets can you fit all 14 of those pieces together to make a larger square? What about 3 complete tangram sets? 4? Can you make an equilateral triangle using a set of tangrams? What types of triangles can you make using all the pieces from  $n$  complete sets of tangrams? Can you design a tangram-like set of polygons that do fit together to form an equilateral triangle?

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<sup>4</sup> Back in the spring of 2011, Jennifer Che of Tiny Urban Kitchen visited Girls' Angle to demonstrate the technique.



# Calendar

Session 27: (all dates in 2020)

September	10	Start of the twenty-seventh session!
	17	
	24	
October	1	
	8	
	15	
	22	
	29	
November	5	
	12	
	19	
	26	Thanksgiving - No meet
December	3	

Session 28: (all dates in 2021)

January	28	Start of the twenty-eighth session!
February	4	
	11	
	18	No meet
	25	
March	4	
	11	
	18	
	25	No meet
April	1	
	8	
	15	
	22	No meet
	29	
May	6	

Girls' Angle has been hosting Math Collaborations at schools and libraries. Math Collaborations are fun math events that can be adapted to a variety of group sizes and skill levels. We will soon have versions available that are designed for remote participation. For more information and testimonials, please visit [www.girlsangle.org/page/math\\_collaborations.html](http://www.girlsangle.org/page/math_collaborations.html).

Girls' Angle can offer custom math classes over the internet for small groups on a wide range of topics. Please inquire for pricing and possibilities. Email: [girlsangle@gmail.com](mailto:girlsangle@gmail.com).

# Girls' Angle: A Math Club for Girls

## Membership Application

**Note: If you plan to attend the club, you only need to fill out the Club Enrollment Form because all the information here is also on that form.**

Applicant's Name: (last) \_\_\_\_\_ (first) \_\_\_\_\_

Parents/Guardians: \_\_\_\_\_

Address (the Bulletin will be sent to this address):

Email:

Home Phone: \_\_\_\_\_ Cell Phone: \_\_\_\_\_

Personal Statement (optional, but strongly encouraged!): Please tell us about your relationship to mathematics. If you don't like math, what don't you like? If you love math, what do you love? What would you like to get out of a Girls' Angle Membership?

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The \$50 rate is for US postal addresses only. **For international rates, contact us before applying.**

Please check all that apply:

- Enclosed is a check for \$50 for a 1-year Girls' Angle Membership.
- I am making a tax free donation.

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to [girlsangle@gmail.com](mailto:girlsangle@gmail.com).



**A Math Club for Girls**

# Girls' Angle Club Enrollment

**Gain confidence in math! Discover how interesting and exciting math can be! Make new friends!**

The club is where our in-person mentoring takes place. At the club, girls work directly with our mentors and members of our Support Network. To join, please fill out and return the Club Enrollment form. Girls' Angle Members receive a significant discount on club attendance fees.

**Who are the Girls' Angle mentors?** Our mentors possess a deep understanding of mathematics and enjoy explaining math to others. The mentors get to know each member as an individual and design custom tailored projects and activities designed to help the member improve at mathematics and develop her thinking abilities. Because we believe learning follows naturally when there is motivation, our mentors work hard to motivate. In order for members to see math as a living, creative subject, at least one mentor is present at every meet who has proven and published original theorems.

**What is the Girls' Angle Support Network?** The Support Network consists of professional women who use math in their work and are eager to show the members how and for what they use math. Each member of the Support Network serves as a role model for the members. Together, they demonstrate that many women today use math to make interesting and important contributions to society.

**What is Community Outreach?** Girls' Angle accepts commissions to solve math problems from members of the community. Our members solve them. We believe that when our members' efforts are actually used in real life, the motivation to learn math increases.

**Who can join?** Ultimately, we hope to open membership to all women. Currently, we are open primarily to girls in grades 5-12. We welcome *all girls* (in grades 5-12) regardless of perceived mathematical ability. There is no entrance test. Whether you love math or suffer from math anxiety, math is worth studying.

**How do I enroll?** You can enroll by filling out and returning the Club Enrollment form.

**How do I pay?** The cost is \$20/meet for members and \$30/meet for nonmembers. Members get an additional 10% discount if they pay in advance for all 12 meets in a session. Girls are welcome to join at any time. The program is individually focused, so the concept of "catching up with the group" doesn't apply.

**Where is Girls' Angle located?** Girls' Angle is based in Cambridge, Massachusetts. For security reasons, only members and their parents/guardian will be given the exact location of the club and its phone number.

**When are the club hours?** Girls' Angle meets Thursdays from 3:45 to 5:45. For calendar details, please visit our website at [www.girlsangle.org/page/calendar.html](http://www.girlsangle.org/page/calendar.html) or send us email.

**Can you describe what the activities at the club will be like?** Girls' Angle activities are tailored to each girl's specific needs. We assess where each girl is mathematically and then design and fashion strategies that will help her develop her mathematical abilities. Everybody learns math differently and what works best for one individual may not work for another. At Girls' Angle, we are very sensitive to individual differences. If you would like to understand this process in more detail, please email us!

**Are donations to Girls' Angle tax deductible?** Yes, Girls' Angle is a 501(c)(3). As a nonprofit, we rely on public support. Join us in the effort to improve math education! Please make your donation out to **Girls' Angle** and send to Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038.

**Who is the Girls' Angle director?** Ken Fan is the director and founder of Girls' Angle. He has a Ph.D. in mathematics from MIT and was a Benjamin Peirce assistant professor of mathematics at Harvard, a member at the Institute for Advanced Study, and a National Science Foundation postdoctoral fellow. In addition, he has designed and taught math enrichment classes at Boston's Museum of Science, worked in the mathematics educational publishing industry, and taught at HCSSiM. Ken has volunteered for Science Club for Girls and worked with girls to build large modular origami projects that were displayed at Boston Children's Museum.

**Who advises the director to ensure that Girls' Angle realizes its goal of helping girls develop their mathematical interests and abilities?** Girls' Angle has a stellar Board of Advisors. They are:

Connie Chow, founder and director of the Exploratory  
Yaim Cooper, Institute for Advanced Study  
Julia Elisenda Grigsby, professor of mathematics, Boston College  
Kay Kirkpatrick, associate professor of mathematics, University of Illinois at Urbana-Champaign  
Grace Lyo, assistant dean and director teaching & learning, Stanford University  
Lauren McGough, postdoctoral fellow, University of Chicago  
Mia Minnes, SEW assistant professor of mathematics, UC San Diego  
Beth O'Sullivan, co-founder of Science Club for Girls.  
Elissa Ozanne, associate professor, University of Utah School of Medicine  
Kathy Paur, Kiva Systems  
Bjorn Poonen, professor of mathematics, MIT  
Liz Simon, graduate student, MIT  
Gigliola Staffilani, professor of mathematics, MIT  
Bianca Viray, associate professor, University of Washington  
Karen Willcox, Director, Oden Institute for Computational Engineering and Sciences, UT Austin  
Lauren Williams, professor of mathematics, Harvard University

**At Girls' Angle, mentors will be selected for their depth of understanding of mathematics as well as their desire to help others learn math. But does it really matter that girls be instructed by people with such a high level understanding of mathematics?** We believe YES, absolutely! One goal of Girls' Angle is to empower girls to be able to tackle *any* field regardless of the level of mathematics required, including fields that involve original research. Over the centuries, the mathematical universe has grown enormously. Without guidance from people who understand a lot of math, the risk is that a student will acquire a very shallow and limited view of mathematics and the importance of various topics will be improperly appreciated. Also, people who have proven original theorems understand what it is like to work on questions for which there is no known answer and for which there might not even be an answer. Much of school mathematics (all the way through college) revolves around math questions with known answers, and most teachers have structured their teaching, whether consciously or not, with the knowledge of the answer in mind. At Girls' Angle, girls will learn strategies and techniques that apply even when no answer is known. In this way, we hope to help girls become solvers of the yet unsolved.

Also, math should not be perceived as the stuff that is done in math class. Instead, math lives and thrives today and can be found all around us. Girls' Angle mentors can show girls how math is relevant to their daily lives and how this math can lead to abstract structures of enormous interest and beauty.

# Girls' Angle: Club Enrollment Form

Applicant's Name: (last) \_\_\_\_\_ (first) \_\_\_\_\_

Parents/Guardians: \_\_\_\_\_

Address: \_\_\_\_\_ Zip Code: \_\_\_\_\_

Home Phone: \_\_\_\_\_ Cell Phone: \_\_\_\_\_ Email: \_\_\_\_\_

Please fill out the information in this box.

**Emergency contact name and number:** \_\_\_\_\_

**Pick Up Info:** For safety reasons, only the following people will be allowed to pick up your daughter. Names:

**Medical Information:** Are there any medical issues or conditions, such as allergies, that you'd like us to know about?

**Photography Release:** Occasionally, photos and videos are taken to document and publicize our program in all media forms. We will not print or use your daughter's name in any way. Do we have permission to use your daughter's image for these purposes? **Yes** **No**

**Eligibility:** Girls roughly in grades 5-12 are welcome. Although we will work hard to include every girl and to communicate with you any issues that may arise, Girls' Angle reserves the discretion to dismiss any girl whose actions are disruptive to club activities.

**Personal Statement (optional, but strongly encouraged!):** We encourage the participant to fill out the optional personal statement on the next page.

**Permission:** I give my daughter permission to participate in Girls' Angle. I have read and understand everything on this registration form and the attached information sheets.

\_\_\_\_\_  
(Parent/Guardian Signature) Date: \_\_\_\_\_

Participant Signature: \_\_\_\_\_

Members: Please choose one.

- Enclosed is \$216 for one session (12 meets)
- I will pay on a per meet basis at \$20/meet.

Nonmembers: Please choose one.

- I will pay on a per meet basis at \$30/meet.
- I'm including \$50 to become a member, and I have selected an item from the left.

I am making a tax free donation.

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to [girlsangle@gmail.com](mailto:girlsangle@gmail.com). Also, please sign and return the Liability Waiver or bring it with you to the first meet.

**Personal Statement (optional, but strongly encouraged!):** This is for the club participant only. How would you describe your relationship to mathematics? What would you like to get out of your Girls' Angle club experience? If you don't like math, please tell us why. If you love math, please tell us what you love about it. If you need more space, please attach another sheet.

**Girls' Angle: A Math Club for Girls  
Liability Waiver**

I, the undersigned parent or guardian of the following minor(s)

\_\_\_\_\_ ,

do hereby consent to my child(ren)'s participation in Girls' Angle and do forever and irrevocably release Girls' Angle and its directors, officers, employees, agents, and volunteers (collectively the "Releasees") from any and all liability, and waive any and all claims, for injury, loss or damage, including attorney's fees, in any way connected with or arising out of my child(ren)'s participation in Girls' Angle, whether or not caused by my child(ren)'s negligence or by any act or omission of Girls' Angle or any of the Releasees. I forever release, acquit, discharge and covenant to hold harmless the Releasees from any and all causes of action and claims on account of, or in any way growing out of, directly or indirectly, my minor child(ren)'s participation in Girls' Angle, including all foreseeable and unforeseeable personal injuries or property damage, further including all claims or rights of action for damages which my minor child(ren) may acquire, either before or after he or she has reached his or her majority, resulting from or connected with his or her participation in Girls' Angle. I agree to indemnify and to hold harmless the Releasees from all claims (in other words, to reimburse the Releasees and to be responsible) for liability, injury, loss, damage or expense, including attorneys' fees (including the cost of defending any claim my child might make, or that might be made on my child(ren)'s behalf, that is released or waived by this paragraph), in any way connected with or arising out of my child(ren)'s participation in the Program.

Signature of applicant/parent: \_\_\_\_\_ Date: \_\_\_\_\_

Print name of applicant/parent: \_\_\_\_\_

Print name(s) of child(ren) in program: \_\_\_\_\_